Assignment for Lecture 12

HIGH-DIMENSIONAL GEOMETRY, SVD AND BEST-FIT SUBSPACE

Lecture Date: 5/20/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn ***before the lecture of 5/27/2026***

1. Show that for any $c \geq 1$, there exist distributions for which Chebyshev's inequality is tight, i.e., $P(|x-\mu| \geq c) = \text{var}[x]/c^2$.

2. Prove that the right singular vector $\mathbf{u}_i$ is the eigenvector of the matrix $\mathbf{A}\mathbf{A}^\top$ corresponding to the eigenvalue σ_i^2.

3. [C] Randomly generate a total 10^3 points on the surface of a sphere in 3 dimensions and 25 dimensions. Create a histogram of all distances between the pairs of points in both cases.