Assignment for Lecture 13

SAMPLING AND RANDOMIZED ALGORITHMS FOR MATRIX MULTIPLICATION

Lecture Date: 5/27/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn ***before the lecture of 6/3/2026***

1. Explain why finding the minimum of $\{1,2,\cdots,n\}$ needs $\mathcal{O}(\ln n)$ space (binary search).

2. How many people must there be before the probability that at least two people have a birthday on October 1 is greater than 50%?

3. [C] As an illustration of the dependence of the quantity $t_{\mathrm{R}} = (\sum_i \sigma_i^2)^2 / \sum_i \sigma_i^4$ on the SV distribution σ_i, consider the following two models: $\sigma_i = 1 - 10^{-3}(i-1)$ and $\sigma_i = \exp[-10^{-2}(i-1)]$ for $i = 1,\cdots,10^3$. Compute numerically the quantity t_{R} of $t \gtrsim t_{\mathrm{R}}$ for the two SV distributions. Plot the singular-value distributions σ_i as functions of i for the two models, and compare their qualitative behaviors.

4. [C] For a squared matrix $\mathbf{A}$ (e.g., with $d = 10^3$) with its elements given by randomly sampling via Unif$[0,2]$, compute the matrices $\mathbf{C}, \mathbf{U}$ and $\mathbf{R}$. Analyze its the performance of the result.