

Assignment for Lecture 14

FAST FOURIER TRANSFORM FOR DATA PROCESSING

Lecture Date: 6/3/2026

“C” denotes for “computational” problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn before the lecture of 6/9/2026

1. Denote t and u are the longitude and the latitude of a point z on the sphere. Prove that one can write it as $z = s e^{it}$ where $s = \tan(\pi/4 + u/2)$.
2. Solve the differential equation $u''(x) - a^2 u(x) + 2a f(x) = 0$ by Fourier transform, where a is a constant.
3. Compute the Fourier transform of the following functions:

$$\text{sinc}(\alpha t), 1/\sqrt{|t|}, \delta(t), \cos \alpha t, \sin \alpha t, t^n, 1/t^n. \quad (1-1)$$

4. [C] A Toeplitz matrix is an $n \times n$ matrix $\mathbf{A} = (a_{ij})$ such that $a_{ij} = a_{i-1, j-1}$ for $i, j = 2, 3, \dots, n$. Give an $\mathcal{O}(n \log n)$ -time algorithm for multiplying a Toeplitz matrix of size n by a vector of length n , and then implement it. Is the sum or product of two Toeplitz matrices necessarily Toeplitz?