

Assignment for Lecture 15

DIFFERENCING SCHEMES FOR PARTIAL DIFFERENTIAL EQUATIONS

Lecture Date: 6/10/2026

“C” denotes for “computational” problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn before the lecture of 6/17/2026

1. A simple algorithm for the 1D convection equation can be obtained from the approximation $[\phi_i^{n+1} - \phi_i^n]/\Delta t \approx -u[\phi_i^n - \phi_{i-1}^n]/\Delta x$. Show analytically that, when $\chi = 1$ is used, the profile can be advected exactly without any numerical error.

2. Show that the leapfrog algorithm for the 1D diffusion equation is

$$\phi_i^{n+1} = \phi_i^{n-1} + \frac{2D\Delta t}{\Delta^2 x} (\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n), \quad (1-1)$$

and then prove that it is always unstable by computing its amplification factor.

3. [C] Solve the equation $\Delta\phi = -xy$ under the boundary conditions $\phi(0) = 0$, $r = \sqrt{x^2 + y^2} \leq 1$.

4. [C] Solve the heat equation on the domain $0 \leq x \leq 1$ and $t \geq 0$ under the initial and boundary conditions $\phi(x, 0) = \sin^2 2\pi x$ and $\phi(0, t) = \phi(1, t) = 0$.