Assignment for Lecture 16

PATH INTEGRAL, SIMULATIONS AND QUANTUM ALGORITHMS

Lecture Date: 6/17/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn ***before the end of this semester***

1. For the forced particle under the constant force f, prove its path integral for the propagator is given by

$$iG(q_{\mathrm{f}},t;q_{\mathrm{i}},0)=\left(\frac{m}{2\pi i\hbar t}\right)^{1/2}\times\exp\left[\frac{i}{\hbar}\left[\frac{m(q_{\mathrm{f}}-q_{\mathrm{i}})^2}{2t}+\frac{1}{2}ft(q_{\mathrm{f}}+q_{\mathrm{i}})-\frac{f^2t^3}{24m}\right]\right]. \tag{1-1}$$

2. Provide a decomposition of the transform

$$\frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & \sqrt{-1} & -1 & -\sqrt{-1}\\ 1 & -1 & 1 & -1\\ 1 & -\sqrt{-1} & -1 & \sqrt{-1}\end{pmatrix} \tag{1-2}$$

   into a product of two-level unitary matrices.

3. [C] Show that if there are x solutions, the probability that Grover's algorithm finds one of them after t iterations is exactly $P_t(x)=\sin^2(2t+1)\vartheta$, where $\cos\vartheta=(1-x/N)^{1/2}$. Rewrite it as

$$P_t(x)=\frac{1}{2}\left[1-\cos\left((2t+1)\arccos\left(1-\frac{2x}{N}\right)\right)\right]. \tag{1-3}$$

   Show that $P_t(x)$ is a polynomial of degree $2t+1$, and verify this result by numerical simulation.