Assignment for Lecture 6

THERMAL MOTION, ANNEALING AND ADVANCED MONTE CARLO SCHEMES

Lecture Date: 4/8/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to `bjcai@fudan.edu.cn` ***before the lecture of 4/15/2026***

1. Use the method of inverse function to generate random variables fulfilling the target distributions $x+x^3$ for $0\le x\le 1$, and $x^2/2+x^5/9$ for $0\le x\le 2$.

2. The ground state of the inverted pendulum could be determined via $\partial U(\beta)/\partial\beta|_{\beta=B}=0$, and if β is small one has $B=[6(1-\theta)]^{1/2}$ where without losing generality the ground state is selected at the right equilibrium position. If one introduces the fluctuation field χ around the ground state by $\beta=B+\chi$, prove that the potential for the fluctuation is

$$\widetilde{U}(\chi)\approx(1-\theta)\chi^2+[(1-\theta)/6]^{1/2}\chi^3+\widetilde{U}_0, \tag{1-1}$$

   where $\widetilde{U}_0=3\theta-3\theta^2/2-3/2$ is a constant term (i.e., independent of the dynamical variable χ). Since the cubic term χ^3 (characterizing the interaction between the field χ) exists, the potential $\widetilde{U}(\chi)$ already has no the symmetry "$\chi\longleftrightarrow-\chi$", demonstrating that the symmetry is broken. Moreover, the first term $(1-\theta)\chi^2$ could be understood as the mass term of χ.

3. [C] Sample the distribution (1-2) with $\nu_1=5$ and $\nu_2=0.1$ using rejection method adopting the Gaussian as the proposal distribution:

$$\widetilde{p}(x)=\sum_{i=1}^{2}\frac{\nu_i}{\sqrt{2\pi\sigma_i^2}}\exp\left(-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right),\ \mu_1=-2,\ \mu_2=3,\ \sigma_1=1,\ \sigma_2=2. \tag{1-2}$$

4. [C] Use the Gaussian as the proposal function to calculate $\int_{-\infty}^{+\infty}x^2 f(x)\mathrm{d}x$, here $f(x)=2^{-1}e^{-|x|}$.