Assignment for Lecture 7

HINDERANCE OF USING $\mathbf{H}^{-1}$, NEWTON-RAPHSON, CONJUGATE GRADIENTS

Lecture Date: 4/15/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to bjcai@fudan.edu.cn ***before the lecture of 4/22/2026***

1. Suppose one has the function $f(x) = (x+1)^2 + (y+3)^2 + 4$. If one starts at the origin, what is the resulting point after one step of Newton's method?

2. Prove that the convergence order of the bisection method is 1.

3. The CG method is applied to a positive definite matrix $\mathbf{A}$ with the result $\|\mathbf{x}^* - \mathbf{x}^{(1)}\|_{\mathbf{A}} = 1$ and $\|\mathbf{x}^* - \mathbf{x}^{(11)}\|_{\mathbf{A}} = 2 \times 2^{-10}$. What bound can you give on $\kappa(\mathbf{A})$?

4. [C] Apply GN and LM algorithms to the minimization of the Rosenbrock function. Fix $a = 1$ and $b = 2$, and $\mathbf{x}^{(0)} = (0.5, 0.5)^\top$. Then use the conjugate gradients.