Assignment for Lecture 8

KALMAN FILTER AS A DATA-DRIVEN OPTIMIZATION METHOD

Lecture Date: 4/22/2026

"C" denotes for "computational" problems, language suggestion: Python/Julia

please include codes and results with analyses for computational problems

please write in pdf format and submit to `bjcai@fudan.edu.cn` ***before the lecture of 4/29/2026***

1. Derive the expression for the Kalman gain in detail, using

$$J(\mathbf{K}_k) = \frac{1}{2}\mathrm{tr}\left(\mathrm{E}\left[\mathbf{e}_k\mathbf{e}_k^\top\right]\right),\quad \mathrm{E}\left[\mathbf{e}_k\mathbf{e}_k^\top\right] = (\vec{1} - \mathbf{K}_k\mathbf{C}_k)\overline{\mathbf{S}}_k(\vec{1} - \mathbf{K}_k\mathbf{C}_k)^\top + \mathbf{K}_k\mathbf{Q}_k\mathbf{K}_k^\top. \tag{1-1}$$

2. Discuss the bias generated by the transform $z = xy$ where x and y are two Gaussian random variables.

3. [C] Consider the discrete-time system

$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} = \begin{pmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{pmatrix} + T\begin{pmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{pmatrix}\left(\begin{pmatrix} v_k \\ \omega_k \end{pmatrix} + \mathbf{w}_k\right),\quad \mathbf{w}_k \sim \mathcal{N}(\vec{0}, \mathbf{R}), \tag{1-2}$$

$$\begin{pmatrix} r_k \\ \phi_k \end{pmatrix} = \begin{pmatrix} \sqrt{x_k^2 + y_k^2} \\ \mathrm{atan2}(-y_k, -x_k) - \theta_k \end{pmatrix} + \mathbf{n}_k,\quad \mathbf{n}_k \sim \mathcal{N}(\vec{0}, \mathbf{Q}), \tag{1-3}$$

which represents a mobile robot moving on the 2D plane and measuring the range and bearing to the origin. Use the EKF to estimate the pose of the mobile robot.

4. [C] In the 1D free-motion problem, the basic formulas are $v(t) = \int_0^t a(t)\mathrm{d}t$, $s(t) = \int_0^t v(t)\mathrm{d}t$, or in discrete form, for example, $v(t+\mathrm{d}t) = v(t) + a(t)\mathrm{d}t$, $s(t+\mathrm{d}t) = s(t) + 2^{-1}(v(t) + v(t+\mathrm{d}t))\mathrm{d}t$, where the acceleration a is composed of two parts, $a(t) = a_0 + \delta_a(t), \delta_a(t) \sim \mathcal{N}(\overline{\delta_a}, \sigma^2_{\delta a})$. If $\overline{\delta_a} = 0$, i.e., $\delta_a(t)$ is a noise term, the velocity has a certain chance to return to its ground truth, but the displacement will always increase. If $\overline{\delta_a} \neq 0$, neither the velocity nor the displacement has any chance to return to its ground truth. Simulate this problem.