

NuSym25, Kobe, 9/8/2025-9/12/2025



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Equation of State of Neutron Star Cores and Maximum Mass for Stable Neutron Stars

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9/11/2025

Collaborated with Dr. Bao-An Li and Dr. Zhen Zhang

Based on:

- (1) Bao-Jun Cai, Bao-An Li and Zhen Zhang, ApJ, 952, 147, 2023
- (2) Bao-Jun Cai, Bao-An Li and Zhen Zhang, PRD, 108, 103041, 2023
- (3) Bao-Jun Cai and Bao-An Li, PRD, 109, 083015, 2024
- (4) Bao-Jun Cai and Bao-An Li, Frontiers, Astron. Space Sci., Volume 11, 2024
- (5) Bao-Jun Cai and Bao-An Li, PRD, 112, 023023, 2025
- (6) Bao-Jun Cai and Bao-An Li, Eur. Phys. A 61, 55, 2025, Review
- (7) Bao-Jun Cai, Bao-An Li and Zhen Zhang, in preparation, 2025

Tolman-Oppenheimer-Volkoff equations

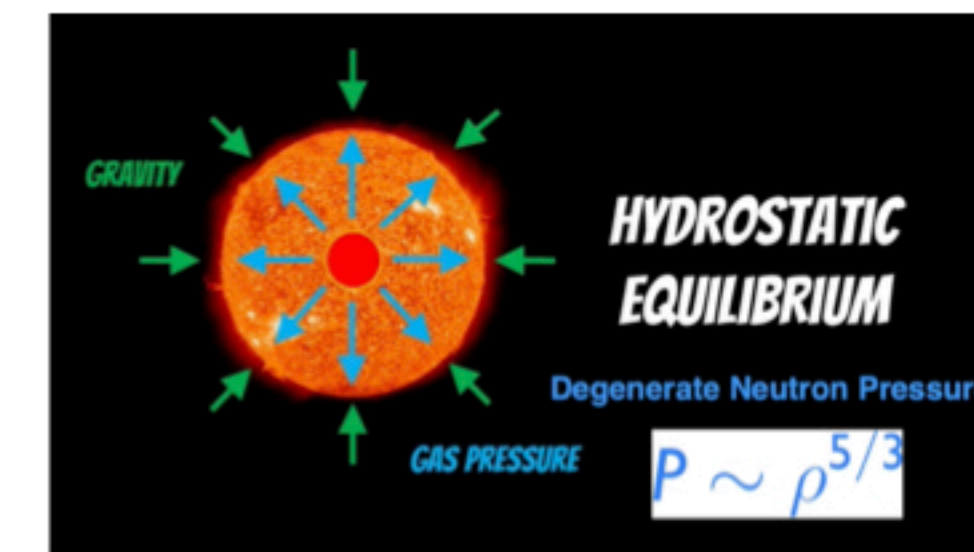


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$$\nabla_\mu T^\mu_\nu = 0$$

$$8\pi G T_{\mu\nu} = R_{\mu\nu} - 2^{-1} g_{\mu\nu} R$$

spherical metric



$$\frac{dP}{dr} = \underbrace{-\frac{GM\epsilon}{r^2}}_{\text{Newtonian}} \underbrace{\left(1 + \frac{P}{\epsilon c^2}\right)}_{\text{matter correction}} \underbrace{\left(1 + \frac{4\pi r^3 P}{Mc^2}\right)}_{\text{matter-geometry coupling}} \underbrace{\left(1 - \frac{2GM}{rc^2}\right)^{-1}}_{\text{geometry correction}}; \quad \underbrace{\frac{dM}{dr} = 4\pi r^2 \epsilon / c^2}_{\text{same as Newtonian}}$$

matter+geometry corrections: $\gg 1$



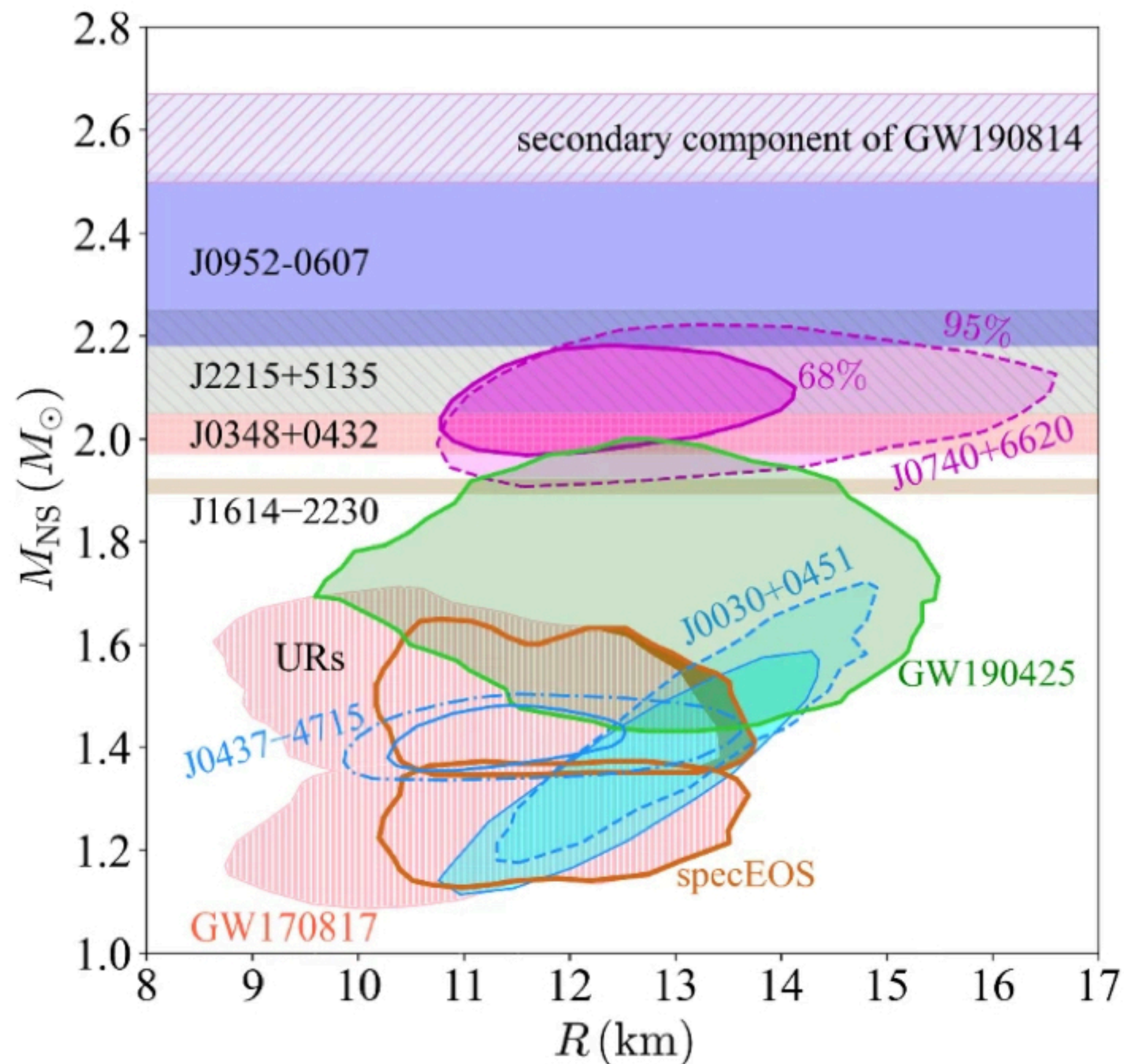
Tolman, PR, 1939; Oppenheimer and Volkoff, PR, 1939

gravity in GR is stronger than Newtonian's

$$R \sim \underbrace{\sqrt{\frac{P}{\epsilon^2}}}_{\text{Newtonian}} \cdot \underbrace{\vartheta(\phi = P/\epsilon)}_{\text{GR correction} < 1}$$

conventionally, one adopts a model for dense matter, solves the general-relativistic structural equations, and infers the properties of the matter

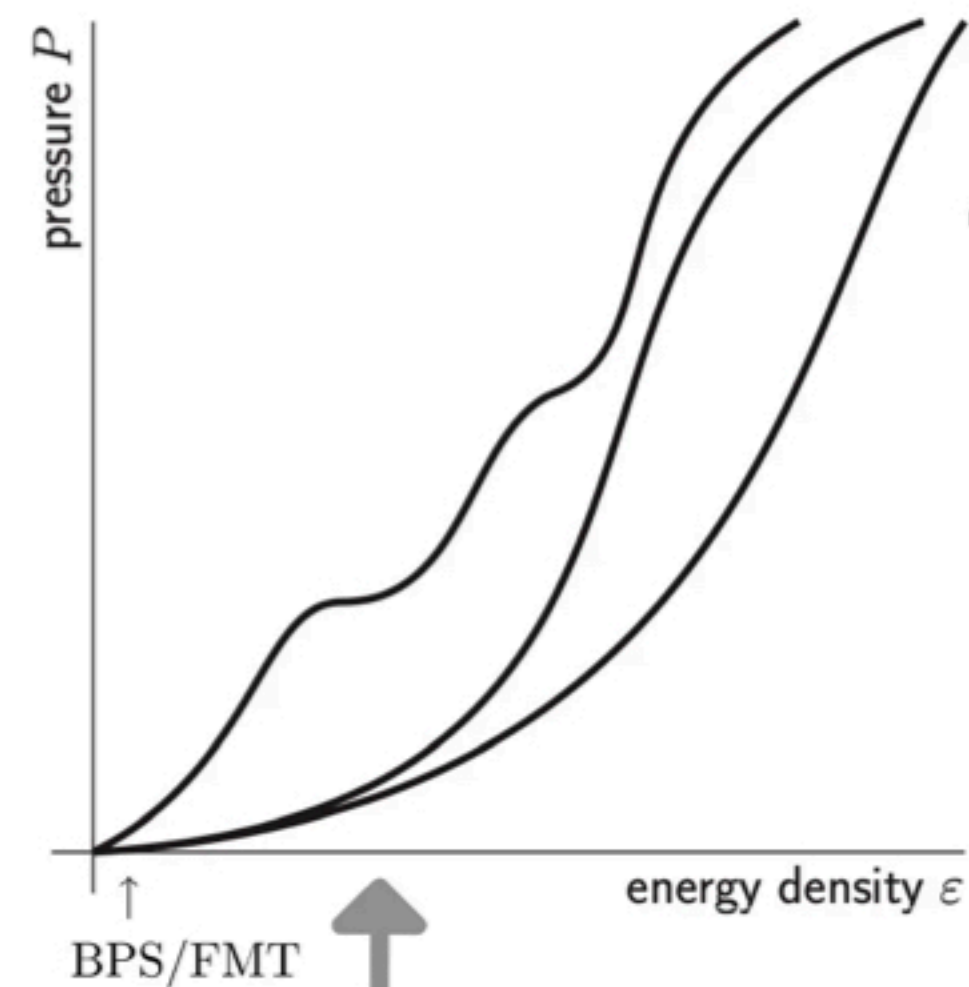
Inverse problem: very difficult



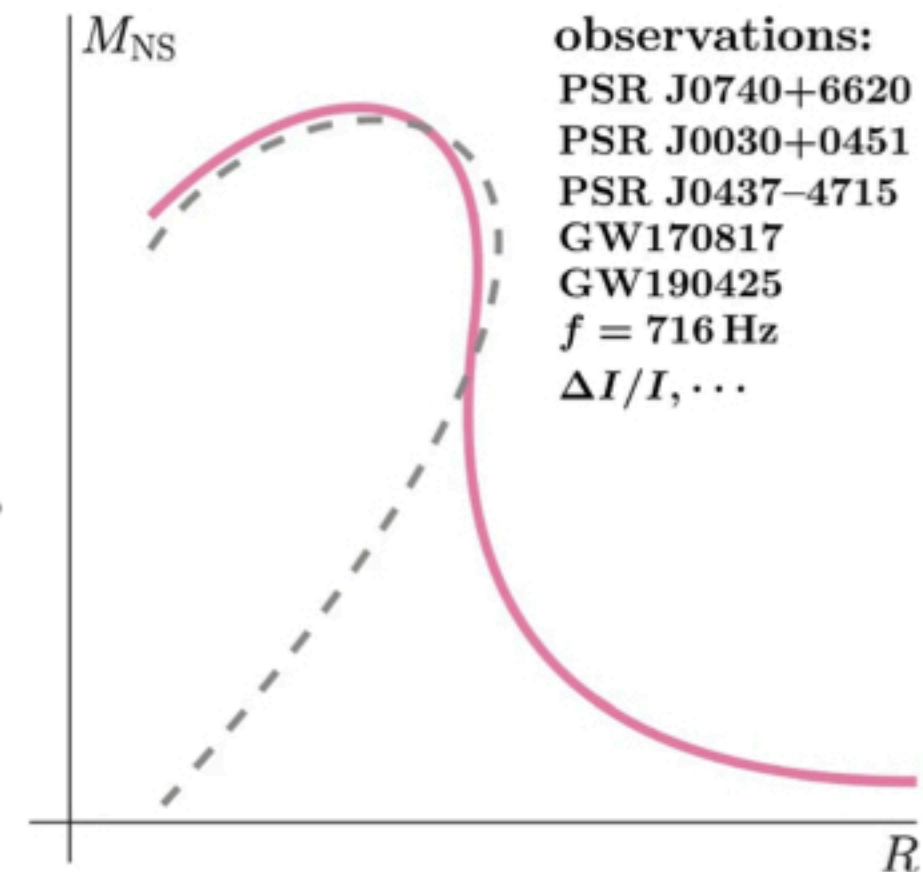
limited observational data on masses and radii of several NSs

BLINDNESS/DEGENERACY OF TOV EQUATIONS

→ one-to-one correspondence →



Tolman-Oppenheimer-Volkoff



observations:
PSR J0740+6620
PSR J0030+0451
PSR J0437-4715
GW170817
GW190425
 $f = 716 \text{ Hz}$
 $\Delta I/I, \dots$

internal quantities

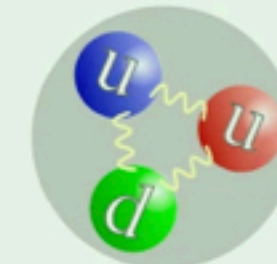
←←← observations
 $(M_{\text{NS}}, R) \rightarrow (P, \epsilon)$ is difficult

model construction for $P(\epsilon)$:

- neutrons/protons
- quarks, gluons
- heavy-ion collision data
- exotics: $\Lambda, \Sigma^{0,\pm}, \Delta^{0,\pm,++}, \dots$
- dark matter (DM), $\dots$
- phase transitions (PTs), $\dots$
- continuous crossover, $\dots$

Example: limitation of the bottom-up approach (EOS of an URFG)

$P = \epsilon/3 \rightarrow \gamma = 1$
TOV Eqs. $\rightarrow \gamma \gtrsim 4/3$ } ↔ inconsistent



Method: perturbation around NS center

length/energy density scale: $Q = \frac{1}{\sqrt{4\pi\epsilon_c}} \sim \mathcal{O}(10 \text{ km}) \sim \text{typical NS radius}$

$$\frac{d\hat{P}}{d\hat{r}} = -\frac{\hat{M}\hat{\epsilon}}{\hat{r}^2} \left(1 + \frac{\hat{P}}{\hat{\epsilon}}\right) \left(1 + \frac{\hat{r}^3\hat{P}}{\hat{M}}\right) \left(1 - \frac{2\hat{M}}{\hat{r}}\right)^{-1}; \quad \frac{d\hat{M}}{d\hat{r}} = \hat{r}^2\hat{\epsilon}; \quad (c = G = 1)$$

$$\hat{P} = X + b_2\hat{r}^2 + b_4\hat{r}^4 + \dots; \hat{\epsilon} = 1 + a_2\hat{r}^2 + a_4\hat{r}^4 + \dots; \hat{M} = c_3\hat{r}^3 + c_5\hat{r}^5 + \dots$$

$$\rightarrow R \sim \nu_c \equiv \frac{X^{1/2}}{\sqrt{\epsilon_c}} \underbrace{\left(\frac{1}{1 + 3X^2 + 4X}\right)^{1/2}}_{\equiv \vartheta(X) < 1}; \quad \underbrace{M_{\text{NS}} \sim R^3 \epsilon_c}_{\text{macroscopic observations}} \sim \underbrace{\Gamma_c \equiv \frac{1}{\sqrt{\epsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X}\right)^{3/2}}_{\text{microscopic combination (EOS)}}$$

(strong gravity reduces the radius)

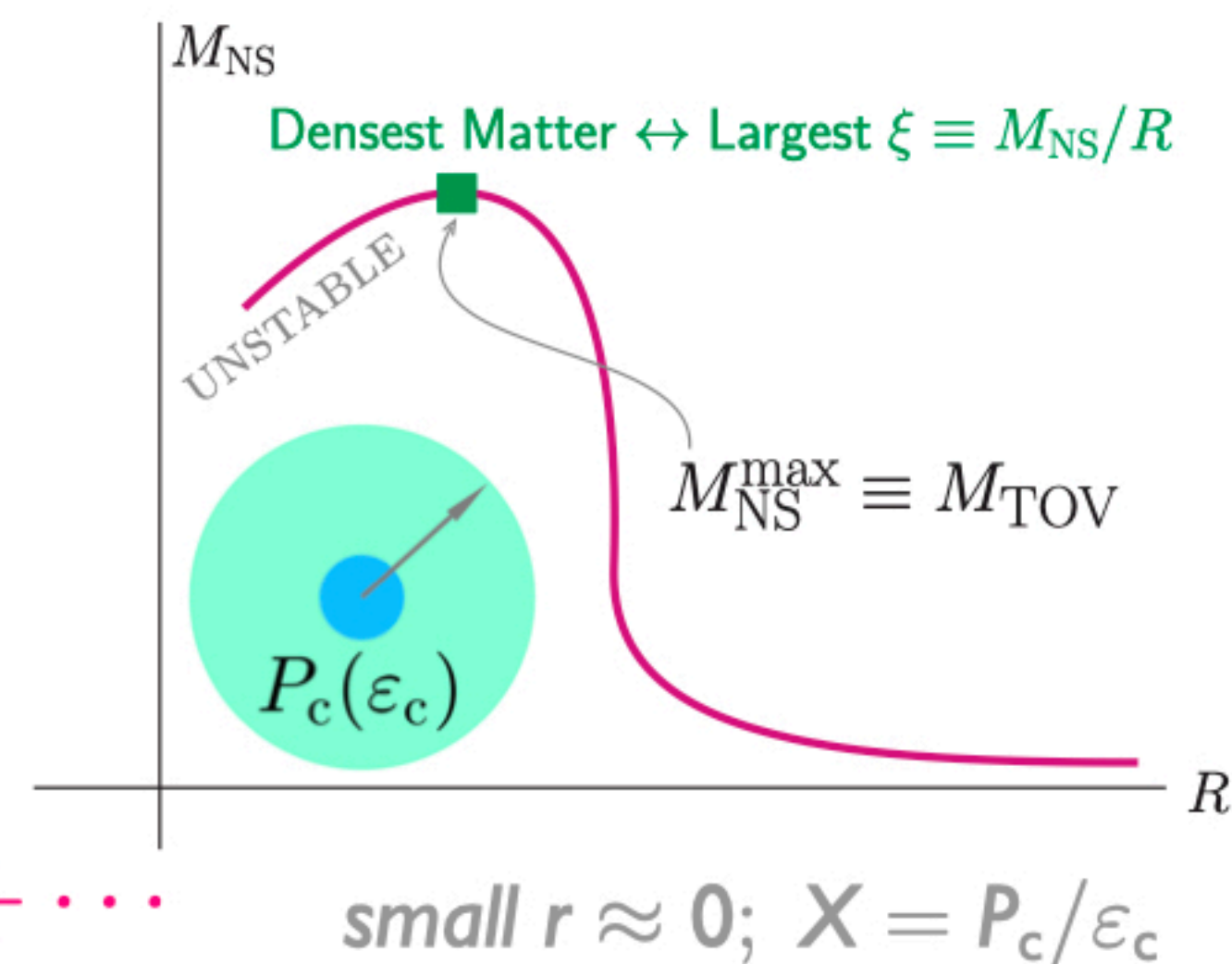
$$s_c^2 \leq 1 \leftrightarrow \phi = P/\epsilon \leq X \lesssim 0.374$$

densest matter $\leftrightarrow$ peak position $\leftrightarrow \frac{dM_{\text{NS}}}{dR} = 0 \rightarrow$

$$s_c^2 = \frac{dP_c}{d\epsilon_c}$$

$$s_c^2 = X \left(1 + \frac{1}{3} \frac{1 + 3X^2 + 4X}{1 - 3X^2}\right) \leftarrow \text{nonlinear}$$

* $P = \epsilon/3$ is inconsistent with NSs



IPAD-TOV: Intrinsic and Perturbative Analyses of Dimensionless TOV Equations

$$\frac{d}{d\hat{r}} \hat{P} = -\frac{\hat{\varepsilon} \hat{M}}{\hat{r}^2} \frac{(1 + \hat{P}/\hat{\varepsilon})(1 + \hat{r}^3 \hat{P}/\hat{M})}{1 - 2\hat{M}/\hat{r}}; \quad \frac{d}{d\hat{r}} \hat{M} = \hat{\varepsilon} \hat{r}^2 \quad E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}(\rho) \delta^2$$

How to perturb?

top-down
(strong-field gravity in GR)
(nuclear model independent)

$$\frac{M_{\text{NS}}}{R} \leftrightarrow \frac{P}{\varepsilon} \leftrightarrow \frac{dP}{d\varepsilon} \leftrightarrow \frac{d \ln P}{d \ln \varepsilon}$$

$$\hat{P} = P/\varepsilon_c \approx X + b_2 \hat{r}^2 + b_4 \hat{r}^4 + \dots$$

$$\hat{\varepsilon} = \varepsilon/\varepsilon_c \approx 1 + a_2 \hat{r}^2 + a_4 \hat{r}^4 + \dots$$

$$\hat{M} \approx \frac{1}{3} \hat{r}^3 + \frac{1}{5} a_2 \hat{r}^5 + \frac{1}{7} a_4 \hat{r}^7 + \dots$$

$$X \equiv P_c/\varepsilon_c, \quad \mu \equiv \hat{\varepsilon} - 1$$

$$\mathcal{U}/\mathcal{U}_c \approx 1 + \sum_{i+j \geq 1} u_{ij} X^i \mu^j$$

(double-element expansion)

numerical

input dense matter EOS

others,...

inference algorithms

astrophysical data

heavy ion collision data

transitions, exotic particles

nuclear structure quantities

nuclear model selected

CEFT (χ EFT): low-density

physical ingredients
mechanisms on EOS

bottom-up

(based on model construction)

- ◆ **core/central EOS**
- ◆ trace anomaly Δ
- ◆ sound speed s^2
- ◆ sharp PTs?
- ◆ conformality?
- ◆ **maximum NS mass**
- ◆ Newtonian limit
- ◆ NS binding energy
- ◆ moments of inertia
- ◆ tidal deformability
- ◆ **slope of MR curve**

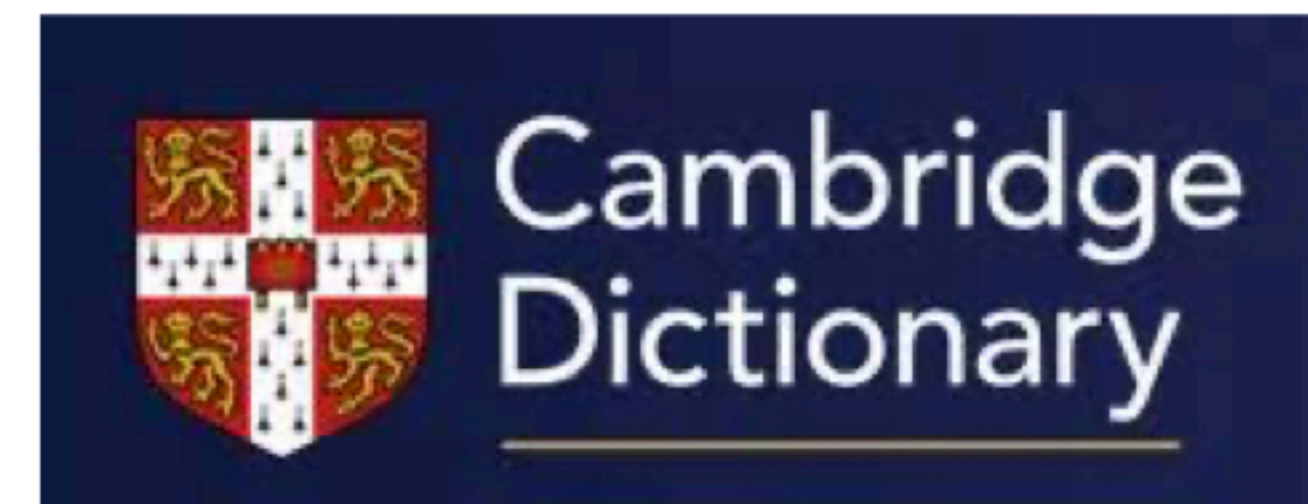
arXiv:2501.18676

★ ultimate goal: core/central EOS in NSs

IPAD-TOV: analogy to Apple's IPAD



Steven Paul Jobs (2/24/1955-10/5/2011)



English meaning of Apple's iPad from Cambridge Dictionary: a brand name for a tablet (aka, small computer) that is controlled by touch rather than having a keyboard.

In our IPAD-TOV approach, certain properties of supra-dense NS matter governed by the TOV equations are probed perturbatively without using a specific input model EOS.

Extracting central NS EOS via similar idea(s)

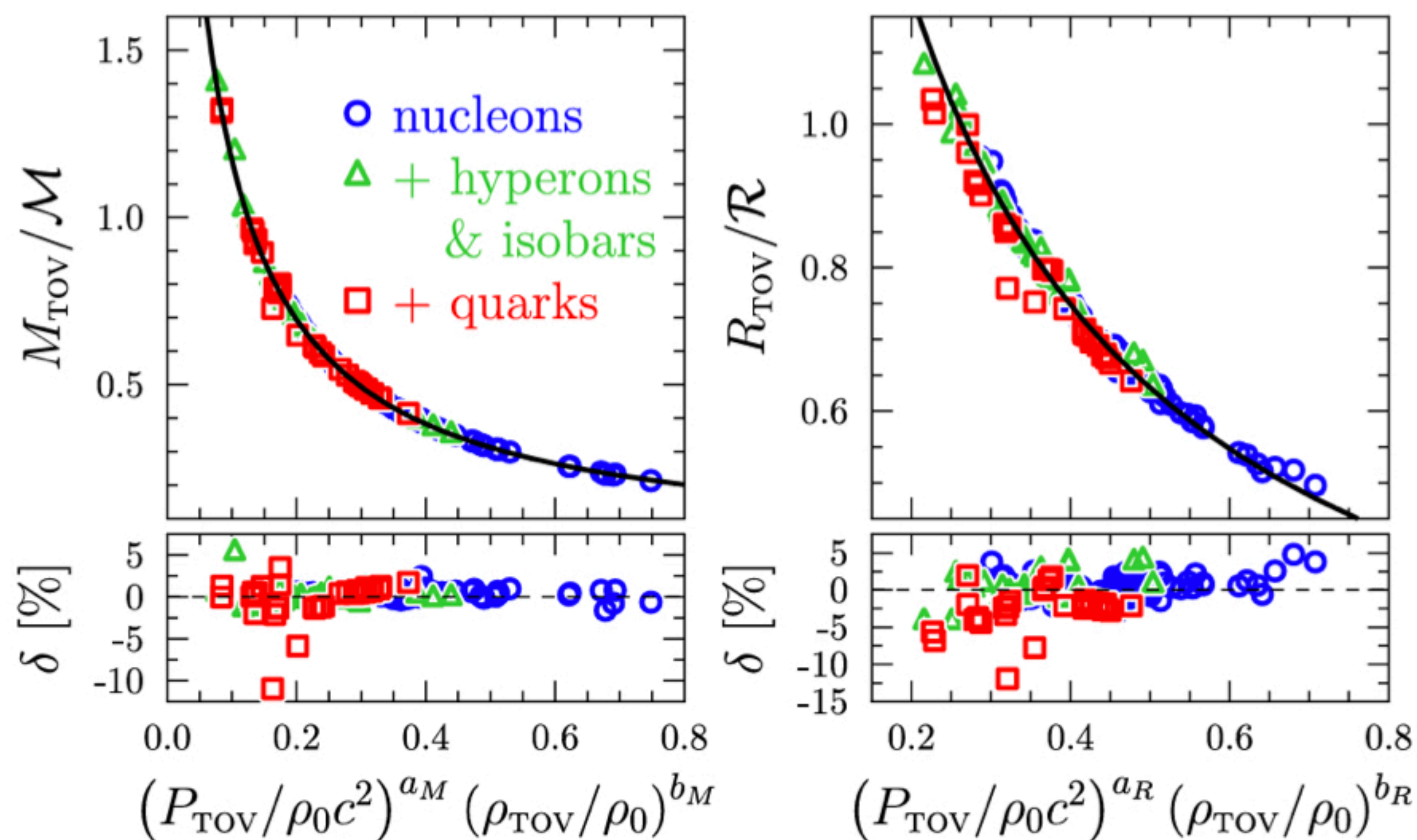


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Ofengeim et al., PRD, 2024

$$M_{\text{NS}}^{\text{max}} = \sqrt{\frac{P_c^3}{\epsilon_c^4} \frac{1}{f_M(P_c, \epsilon_c)}}, \quad R_{\text{NS}}^{\text{max}} = \sqrt{\frac{P_c}{\epsilon_c^2} \frac{1}{f_R(P_c, \epsilon_c)}}$$

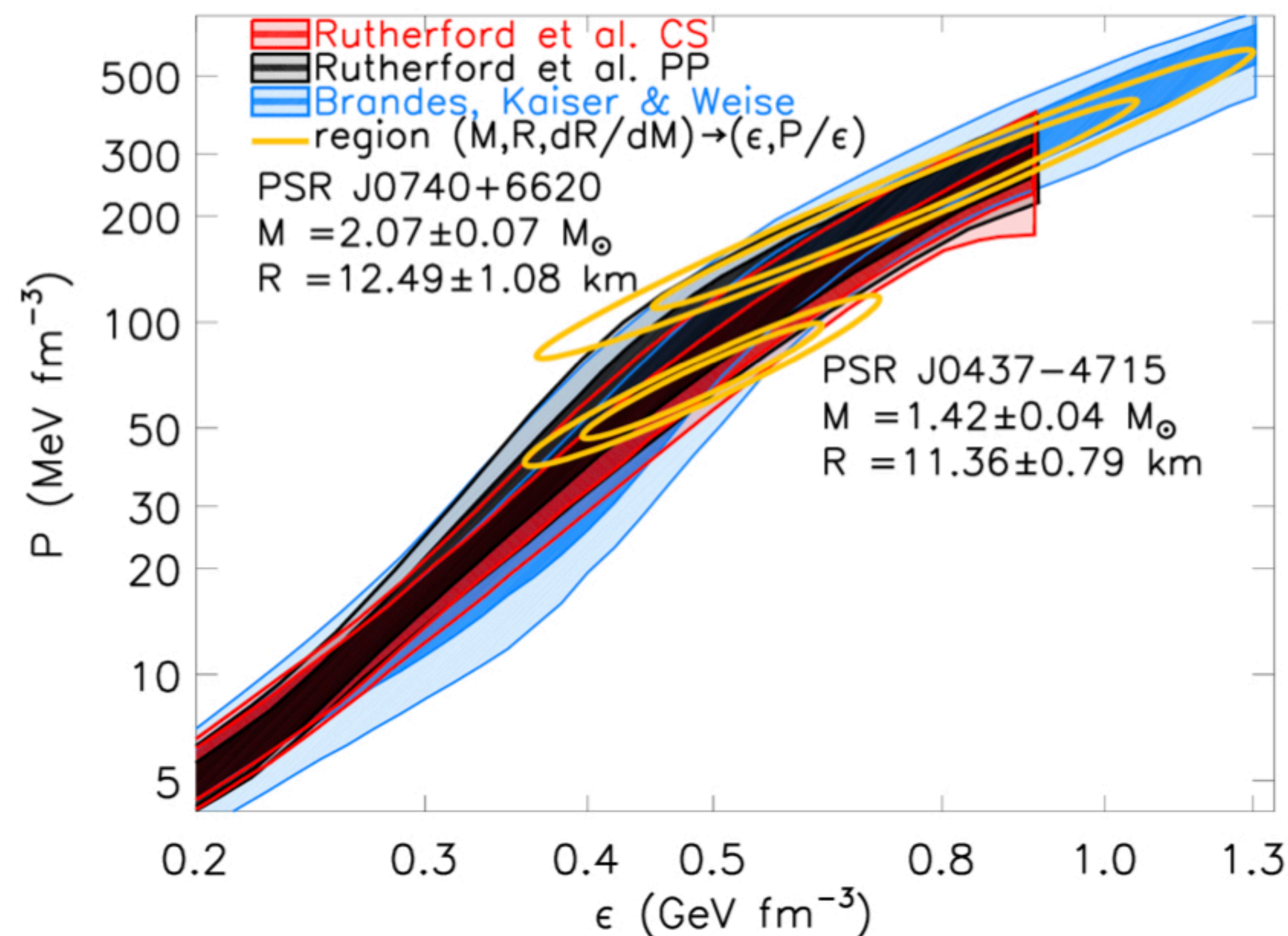
f_M, f_R : fitting functions with 3 parameters for each



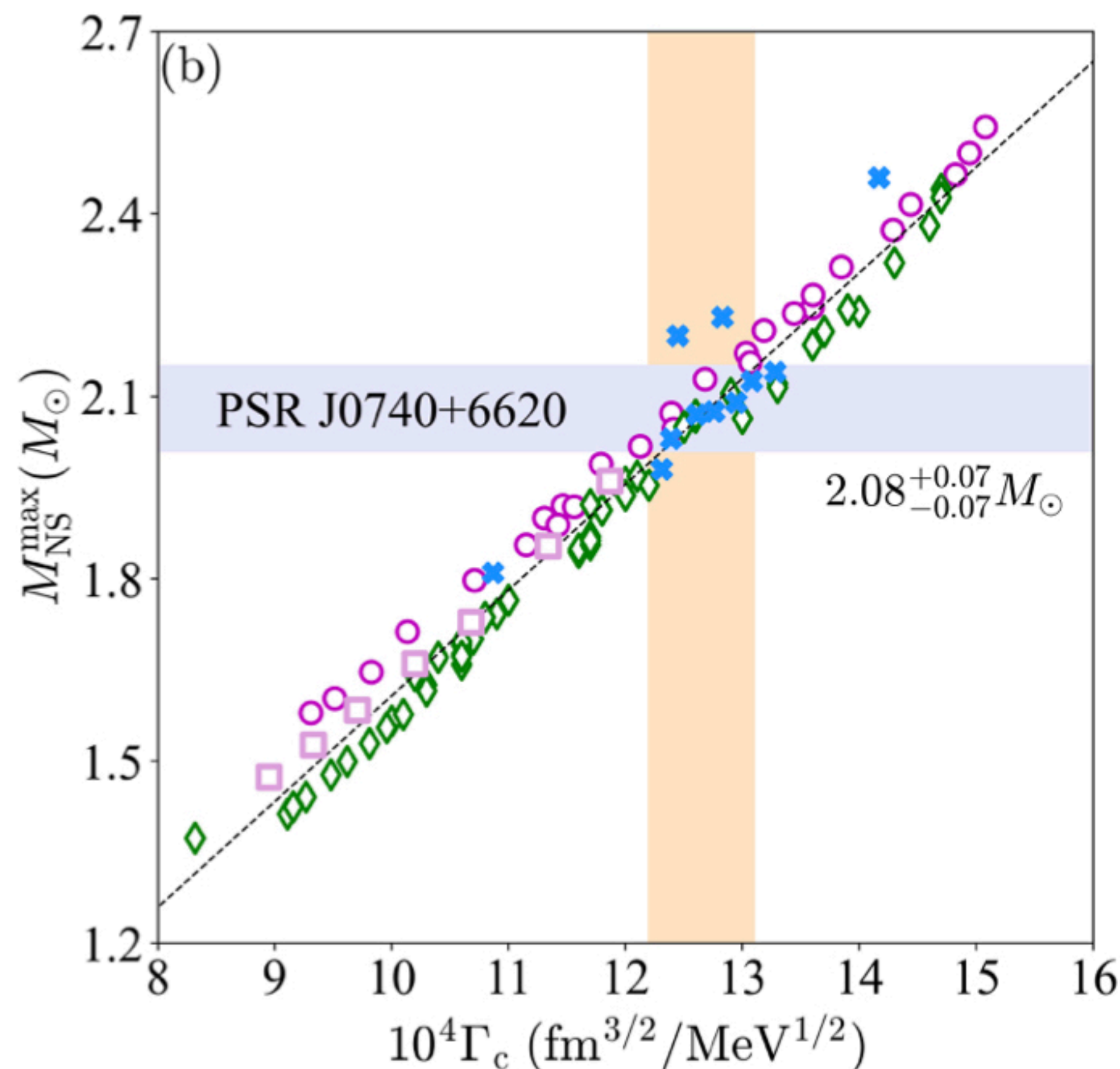
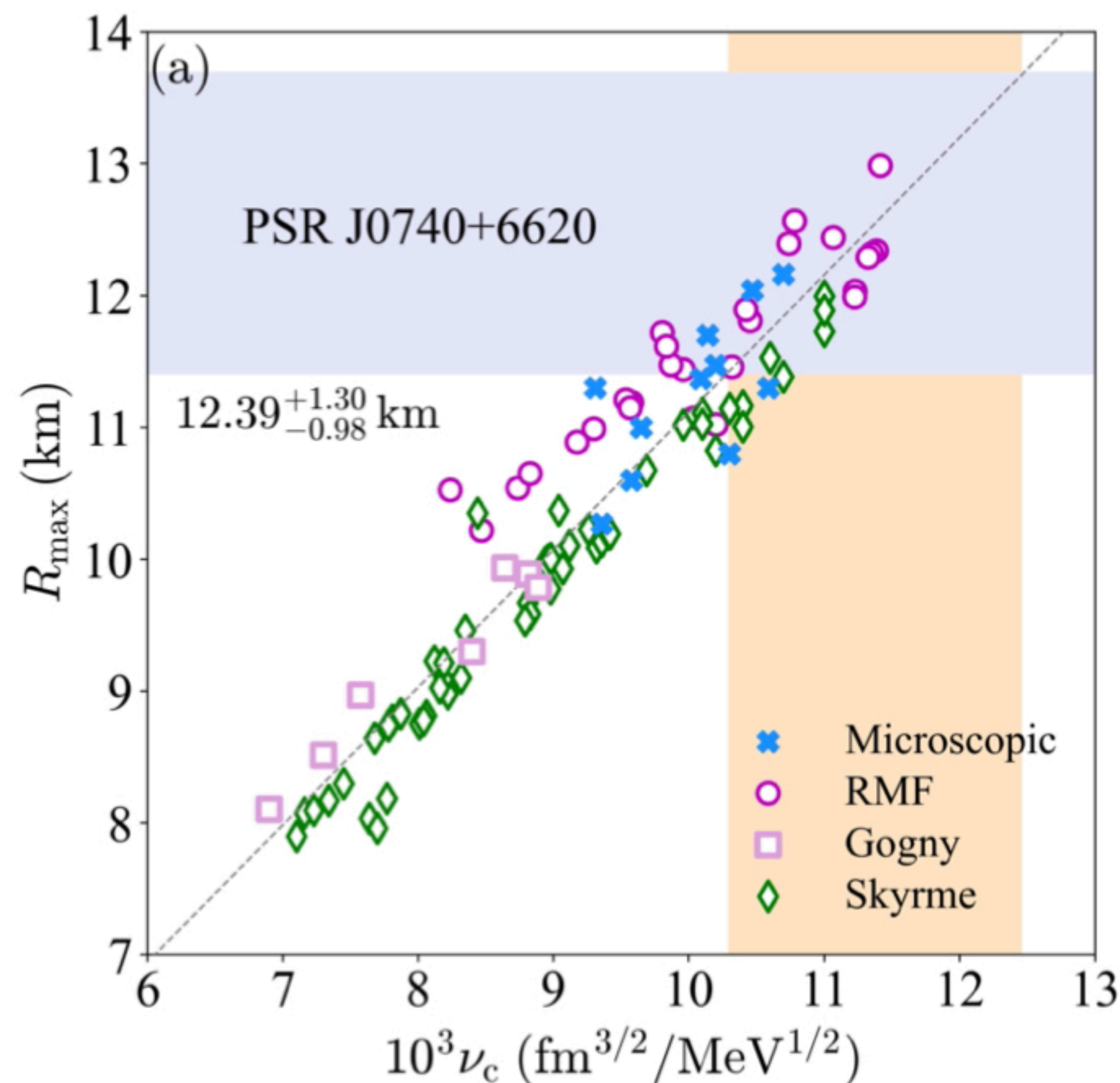
Sun and Lattimer, ApJ, 2025

power law fitting: $\mathcal{P} = A_{\mathcal{P}} P_c^{B_{\mathcal{P}}} \epsilon_c^{C_{\mathcal{P}}}$ for $\mathcal{P} = M_{\text{NS}}^{\text{max}}, R_{\text{NS}}^{\text{max}}$

$A_{\mathcal{P}}, B_{\mathcal{P}}, C_{\mathcal{P}}$: fitting parameters, RMS accuracy $\lesssim 0.5\%$



Mass and radius scalings from IPAD-TOV



EOS lists (#=104):

- * Skyrme, Gogny, RMF
- * APR (2N+3N)
- * DBHF (MPAI, ENG)
- * chiral mean fields (CMFs)
- * hyperons (DD2, H4)
- * hybrid (ALF2)
- * strange quark stars (SQM3)

$$\nu_c = \frac{1}{\sqrt{\varepsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{1/2}$$

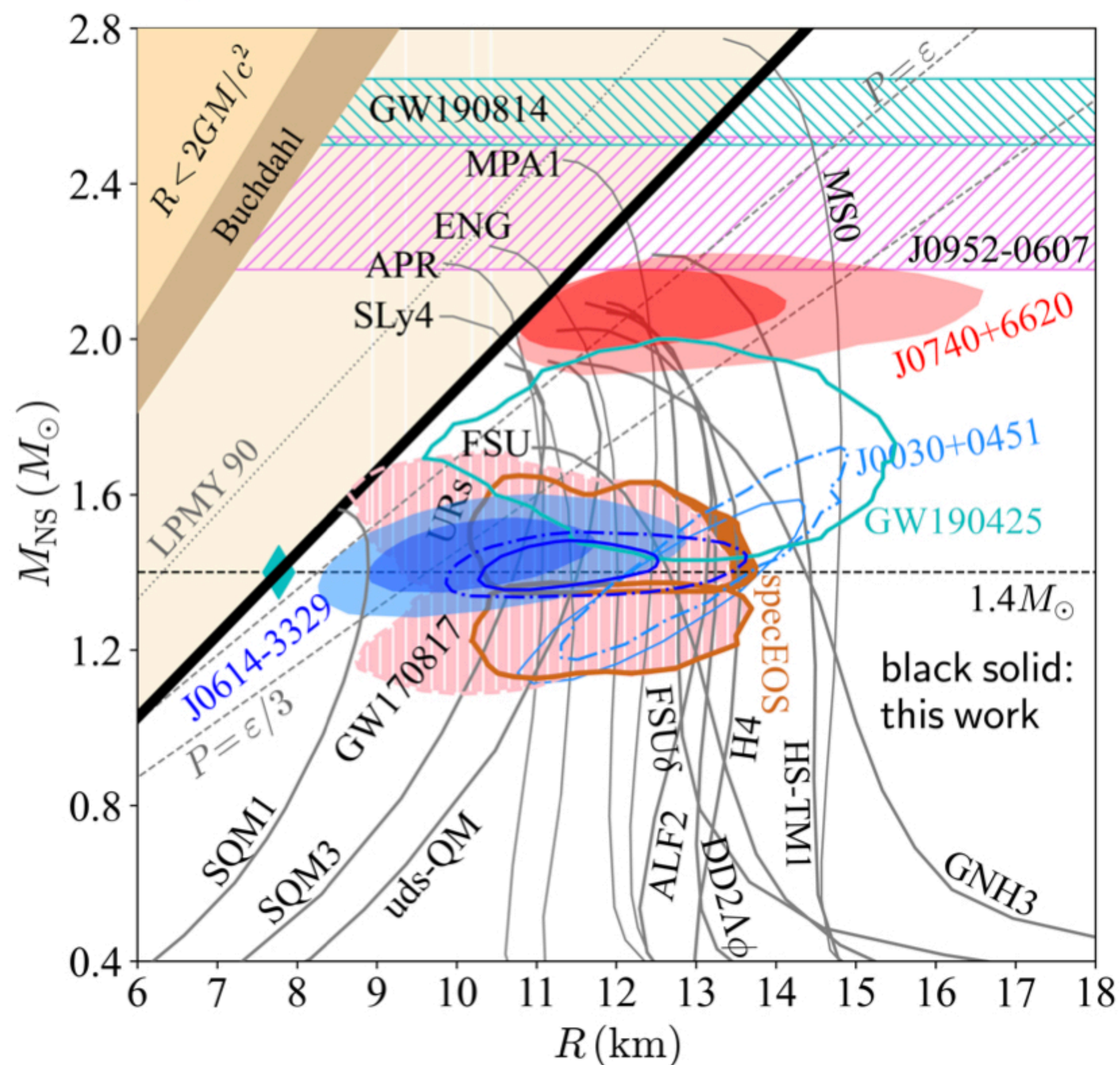
$$X = P_c / \varepsilon_c \rightarrow P_c(\varepsilon_c)$$

$$\Gamma_c = \frac{1}{\sqrt{\varepsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}$$

Can a massive NS have a small radius $\lesssim 10\text{km}$?



J0614: from Sebastien Guillot's talk

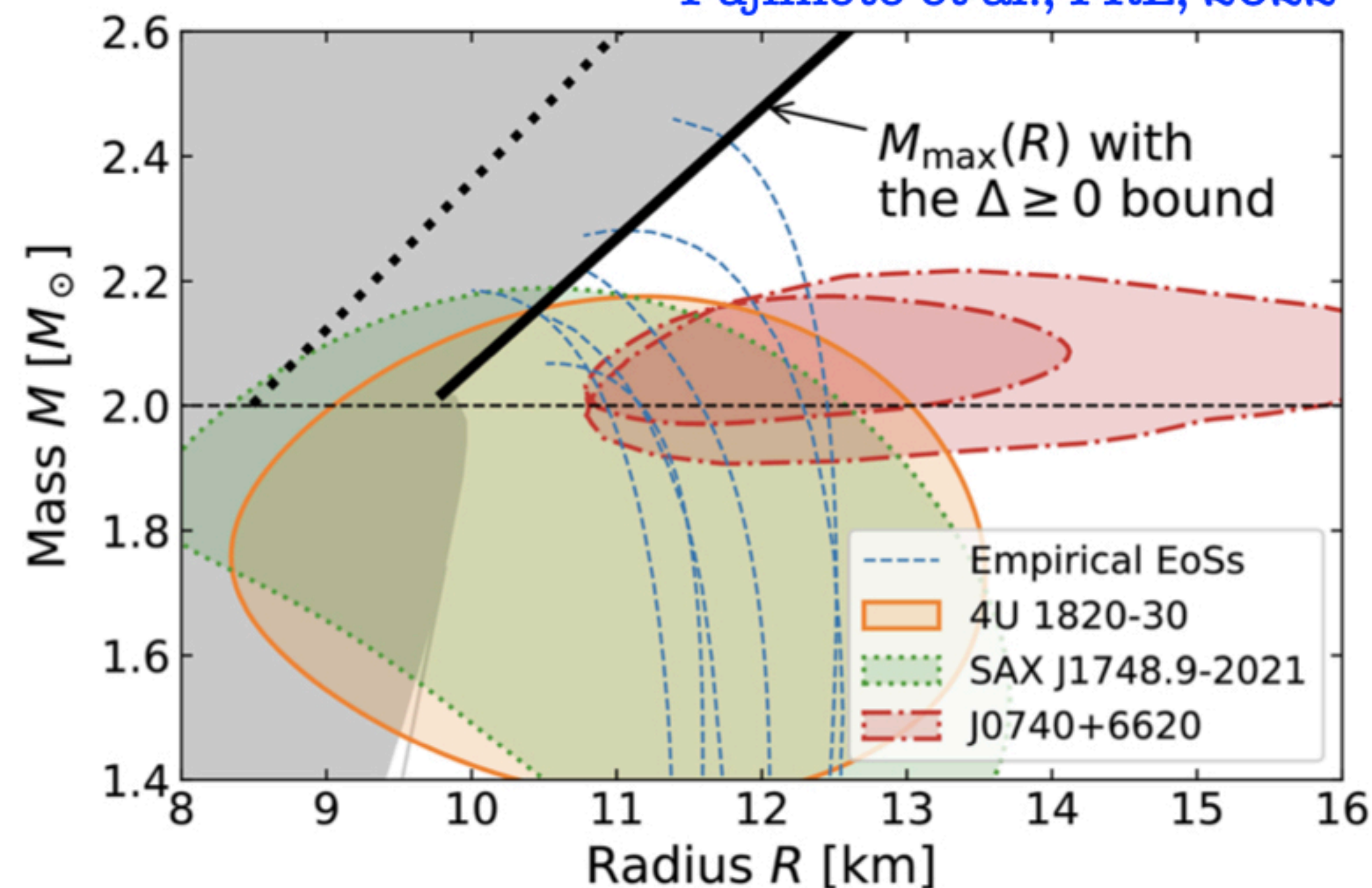


this boundary SHOULD be checked
by observational NS data/GW signals

$$\leftarrow R_{\text{max}}/\text{km} \gtrsim 4.73 M_{\text{NS}}^{\text{max}}/M_{\odot} + 1.14$$

$$M_{\text{NS}}^{\text{max}}/M_{\odot} \approx 2 \rightarrow R_{\text{max}} \gtrsim 10.6 \text{ km}$$

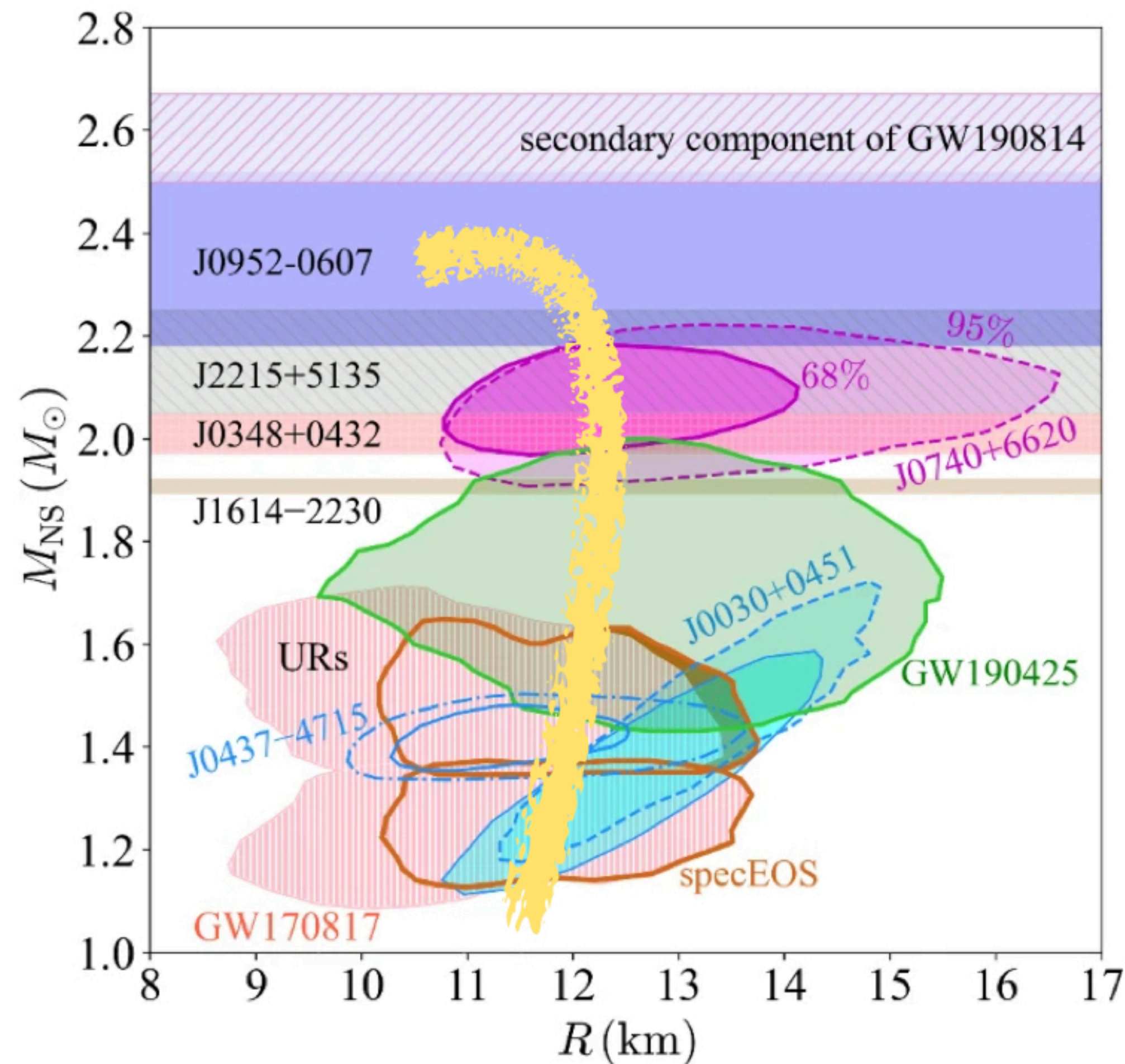
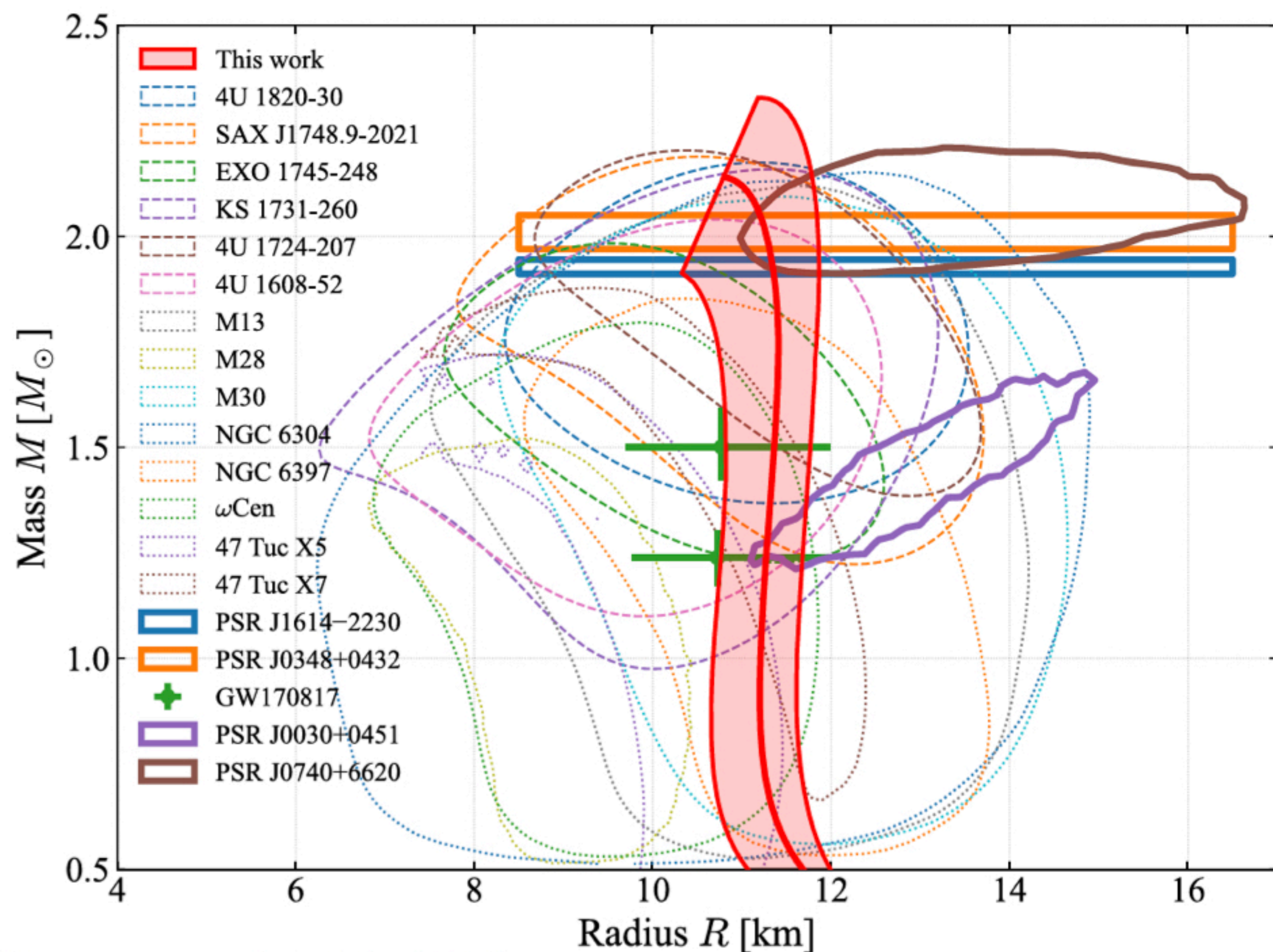
Fujimoto et al., PRL, 2022



The empirical vertical shape for NS M-R curve



empirical fact: M-R curve for NSs with $M_{\text{NS}}/M_{\odot} \approx 1.2\text{-}2.2$ is roughly **vertical**



Understanding the slope of NS M-R curve

$$\ell \equiv \frac{dM_{\text{NS}}}{dR} \approx \overbrace{\frac{3\Psi}{\Psi - 2} \frac{M_{\text{NS}}}{R}}^{\text{positive or negative}}, \quad \Psi \equiv 2 \underbrace{\frac{d \ln M_{\text{NS}}}{d \ln \varepsilon_c}}_{\text{positive}}$$

$\Psi \approx 2-3$ for these NSs with $M_{\text{NS}}/M_{\odot} \approx 1.2-2.2$

$\Psi = 2 \rightarrow \text{linear growth } M_{\text{NS}} \sim \varepsilon_c$

monotonic or
positive-definite

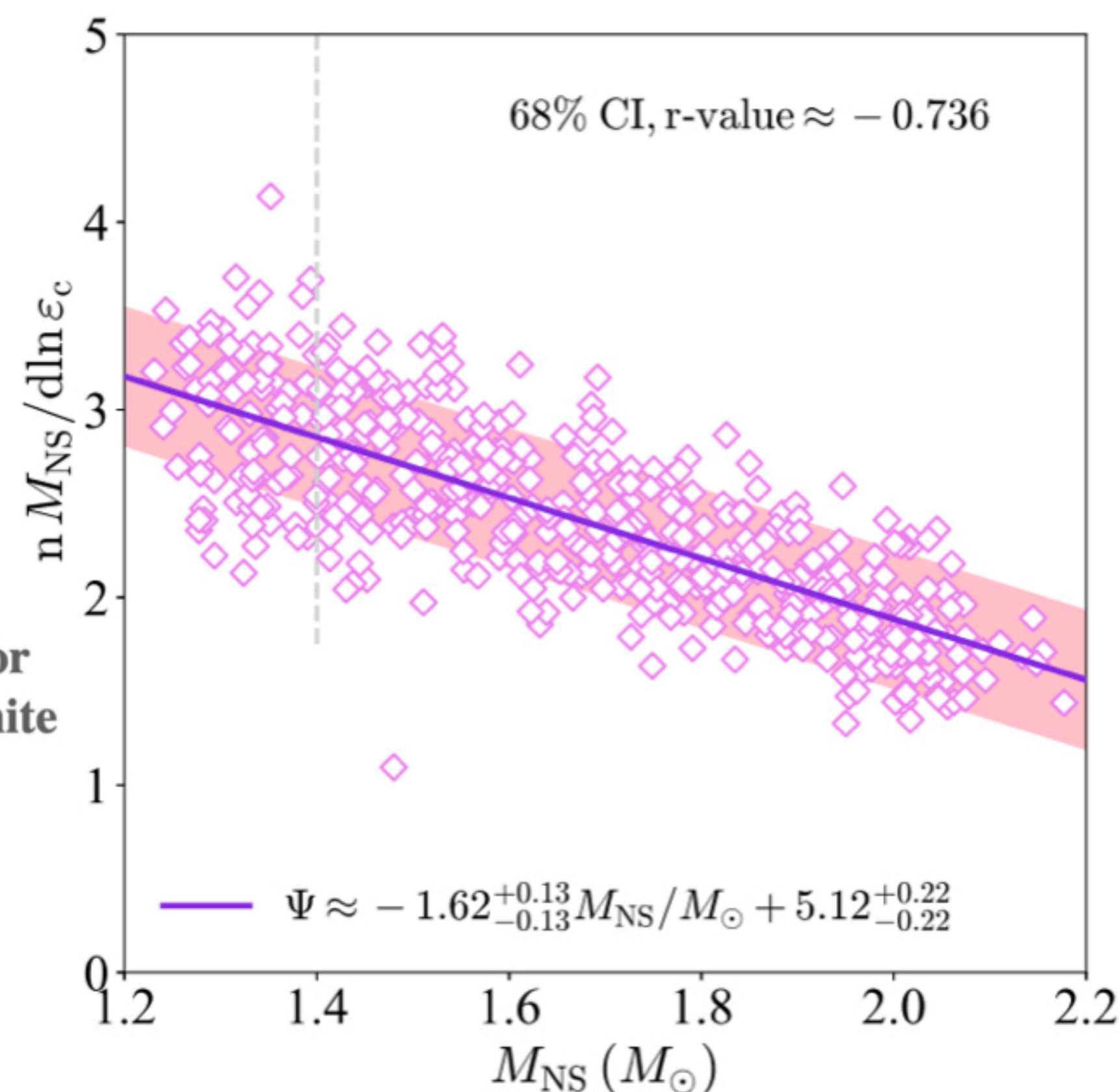
$$\phi = \frac{P}{\varepsilon} \xrightarrow{\text{denseness and stiffness}} s^2 = \frac{dP}{d\varepsilon}$$

non-monotonic or
non positive-definite

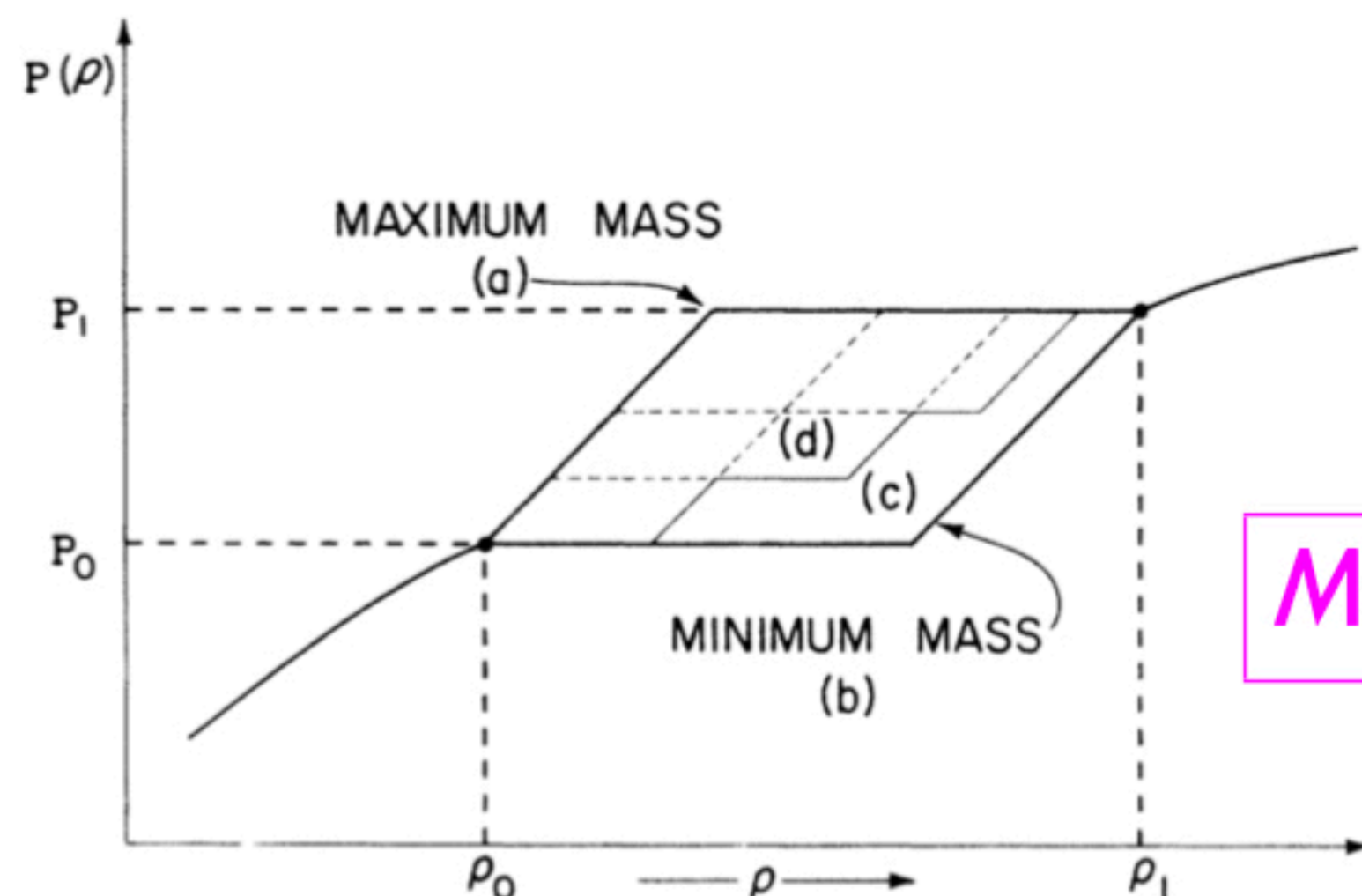
$$\xi = \frac{M_{\text{NS}}}{R} \xrightarrow{\text{compactness and slope of NS M-R curve}} \frac{dM_{\text{NS}}}{dR} \approx \frac{3M_{\text{NS}}/R}{1 - 2/\Psi}$$

(1) light NSs: $\Psi \gtrsim 2 \rightarrow \ell > 0$

(2) massive NSs: $\Psi \lesssim 2 \rightarrow \ell < 0$



Stable NSs have a maximum mass: TOV mass



stability condition: $s^2 \geq 0$ ✓

causality principle: $s^2 \leq 1$ ✓

unrealistic “stiff EOS”: $P \approx \varepsilon$??

NSs → BHs

Fishbach, Science, 2024

$$M_{\text{TOV}} \lesssim 3.2M_{\odot}$$

Masses of neutron stars and black holes

Neutron stars and stellar-mass black holes have been observed in electromagnetic events (pulsars, x-ray binaries, and isolated black holes), as well as with certain gravitational-wave events. The gray band represents the uncertain mass limit separating neutron stars and black holes. The companion to pulsar PSR J0514–4002E is represented by the larger circle, with its 95% uncertainty interval.

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PHYSICAL REVIEW LETTERS

11 FEBRUARY 1974

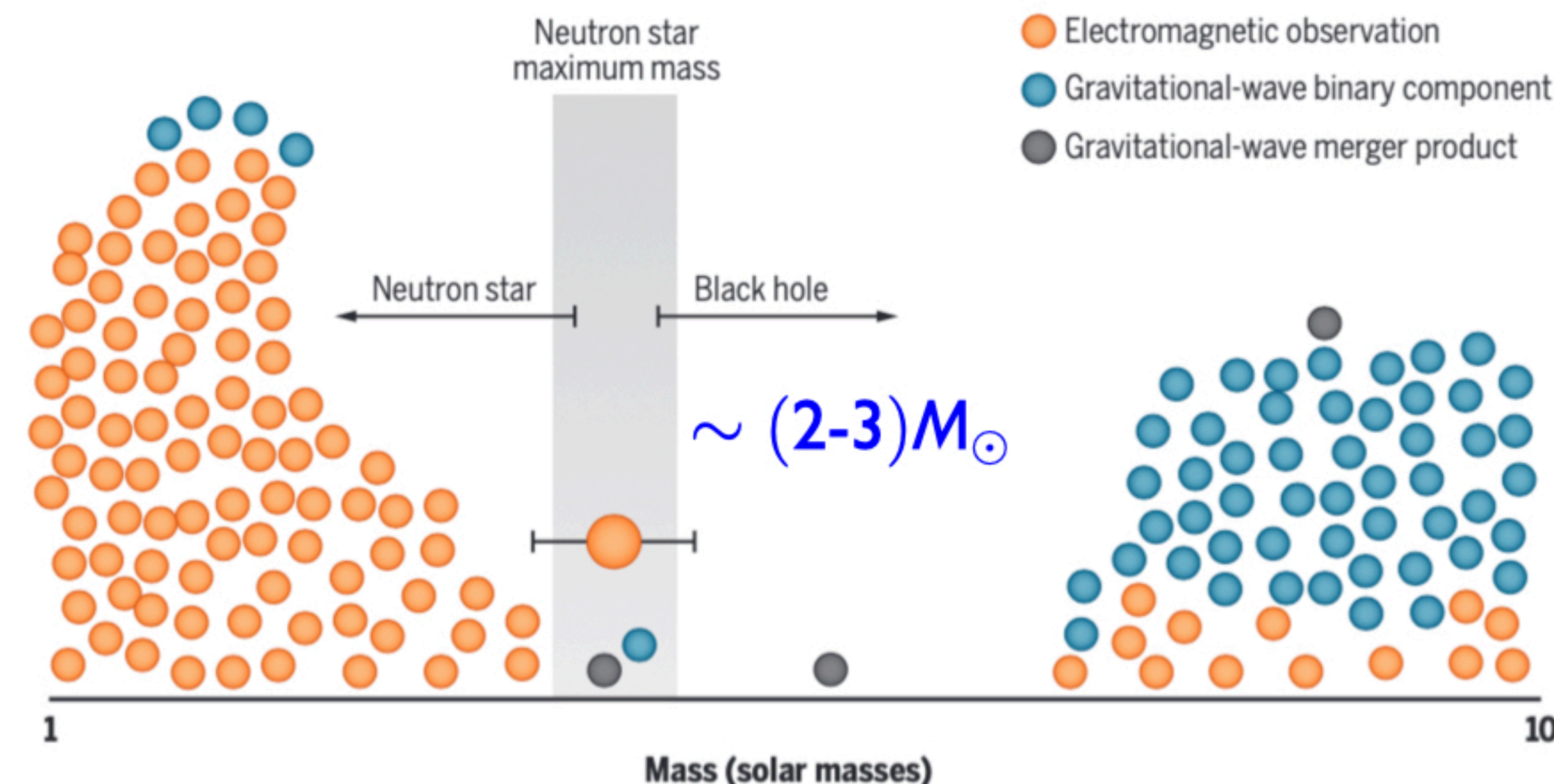
Maximum Mass of a Neutron Star*

Clifford E. Rhoades, Jr.,† and Remo Ruffini

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 30 October 1972)

On the basis of Einstein's theory of relativity, the principle of causality, and Le Chatelier's principle, it is here established that the maximum mass of the equilibrium configuration of a neutron star cannot be larger than $3.2M_{\odot}$. The extremal principle given here applies as well when the equation of state of matter is unknown in a limited range of densities. The absolute maximum mass of a neutron star provides a decisive method of observationally distinguishing neutron stars from black holes.



Existence of TOV mass: the physical intuition

$$X \lesssim 0.374$$

(1) self-gravitating: $M_{\text{NS}} \sim \frac{1}{\sqrt{\epsilon_c}} \overbrace{\left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}}^{\text{upper bounded}} \rightarrow \epsilon_c \sim M_{\text{NS}}^{-2} \quad Y_+$

(2) stability condition: $dM_{\text{NS}}/d\epsilon_c \geq 0 \rightarrow \epsilon_c$ increases with $M_{\text{NS}} \quad Y_-$

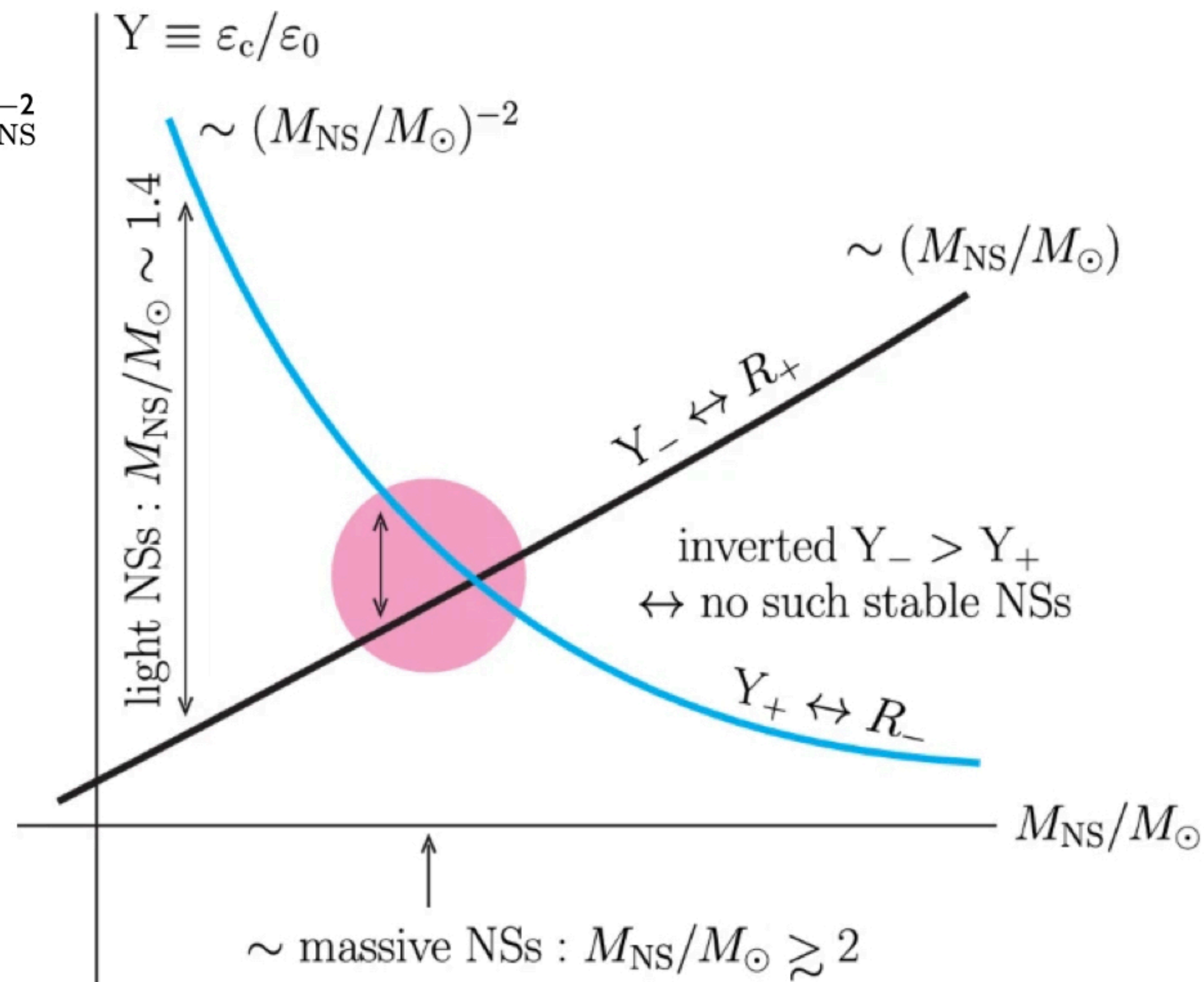
$$\frac{\epsilon_c}{\epsilon_0} \lesssim \frac{21.71}{(M_{\text{NS}}/M_{\odot} + 0.08)^2} \equiv Y_+ \leftrightarrow R_-$$

canonical NSs $M_{\text{NS}}/M_{\odot} \approx 1.4$: $\epsilon_c/\epsilon_0 \lesssim 9.9$

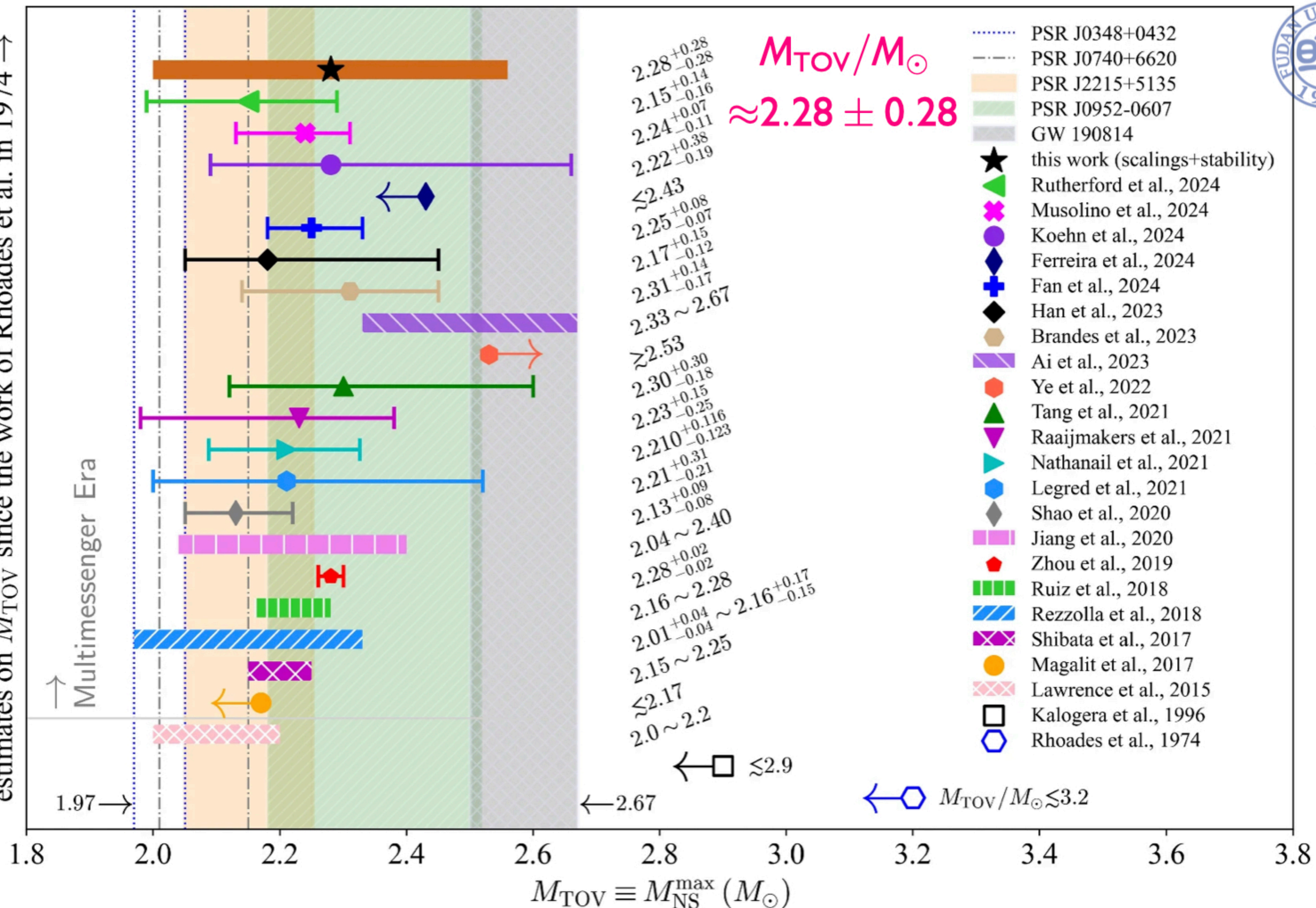
massive NSs $M_{\text{NS}}/M_{\odot} \approx 3.0$: $\epsilon_c/\epsilon_0 \lesssim 2.3$

$$\epsilon_0 \approx 150 \text{ MeV/fm}^3$$

strange?!



estimates on M_{TOV} since the work of Rhoades et al. in 1974 $\rightarrow$



consistent
but no surprise

Summary of my talk

- (1) EOS of NS cores directly from mass/radius observations, e.g., $P/\varepsilon \lesssim 0.374$
- (2) empirical “vertical” shape of M-R curve $\leftrightarrow$ linear growth of $M_{\text{NS}} \sim \varepsilon_c$
- (3) self-gravitating + stability condition $\rightarrow M_{\text{TOV}}/M_{\odot} \approx 2.28 \pm 0.28$

Thank you for your attention!

Stellar radius scaling: dimensional analysis



$$\underbrace{\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}}_{\text{Newtonian gravity}} \rightarrow P \sim Gr^2\epsilon^2 \text{ since } M \sim \epsilon r^3 \rightarrow R \sim \sqrt{\frac{P}{\epsilon^2}} \sim \sqrt{\frac{P}{\epsilon}} \sim \sqrt{\frac{\phi}{\epsilon}}$$

$\phi = P/\epsilon \sim \text{dimensionless}$

as energy density increases:

unstable: $\frac{dM_{\text{NS}}}{d\epsilon_c} < 0$

(1) **radius decreases**; (2) **while mass increases**

compactness : $\xi = GM_{\text{NS}}/Rc^2 = M_{\text{NS}}/R$

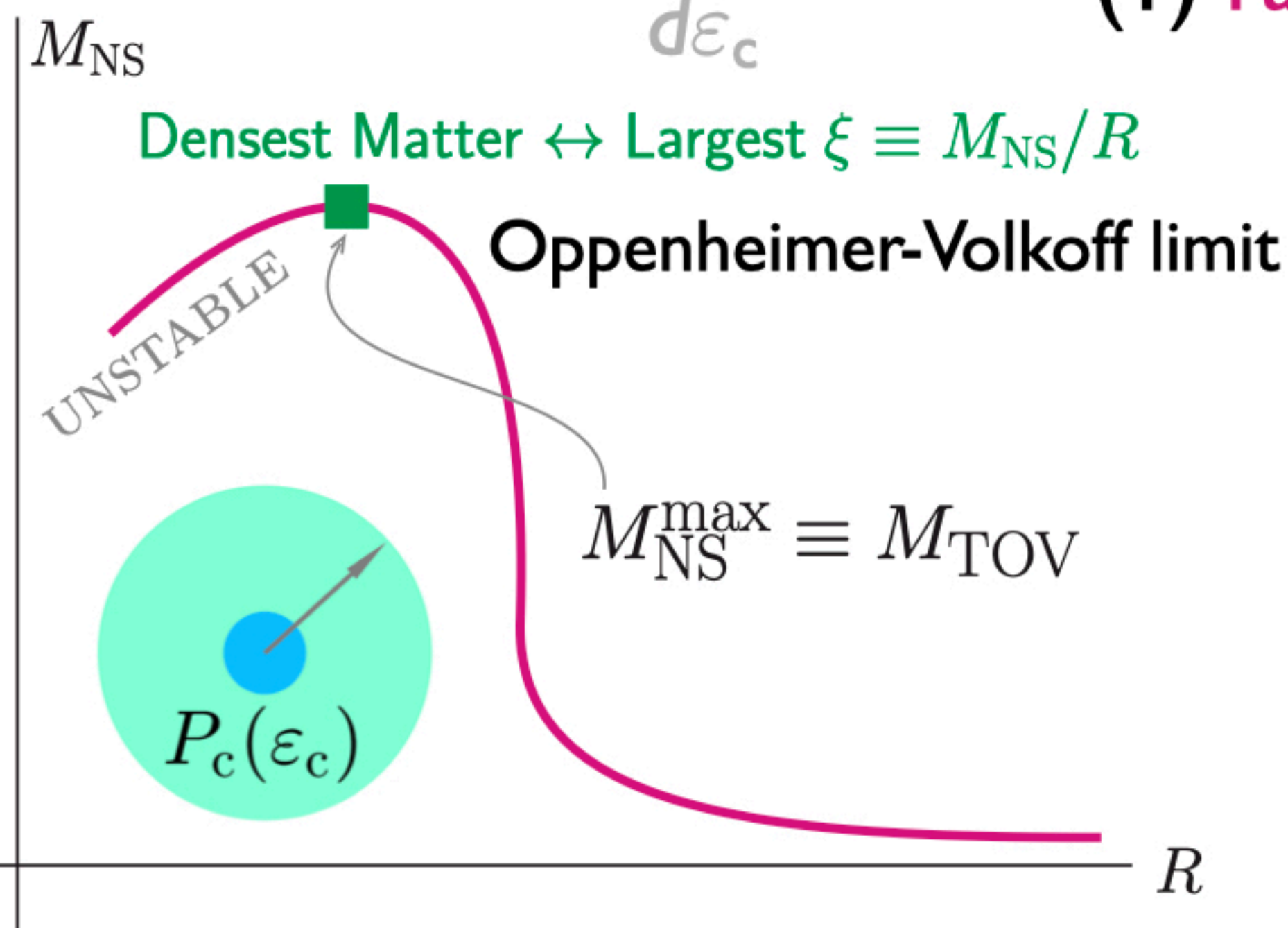
larger $\xi \leftrightarrow$ denser; densest $\leftrightarrow$ maximum configuration

$M_{\odot} \approx 1.5 \text{ km} \rightarrow \xi_{\text{NS}} \sim \mathcal{O}(0.1)$

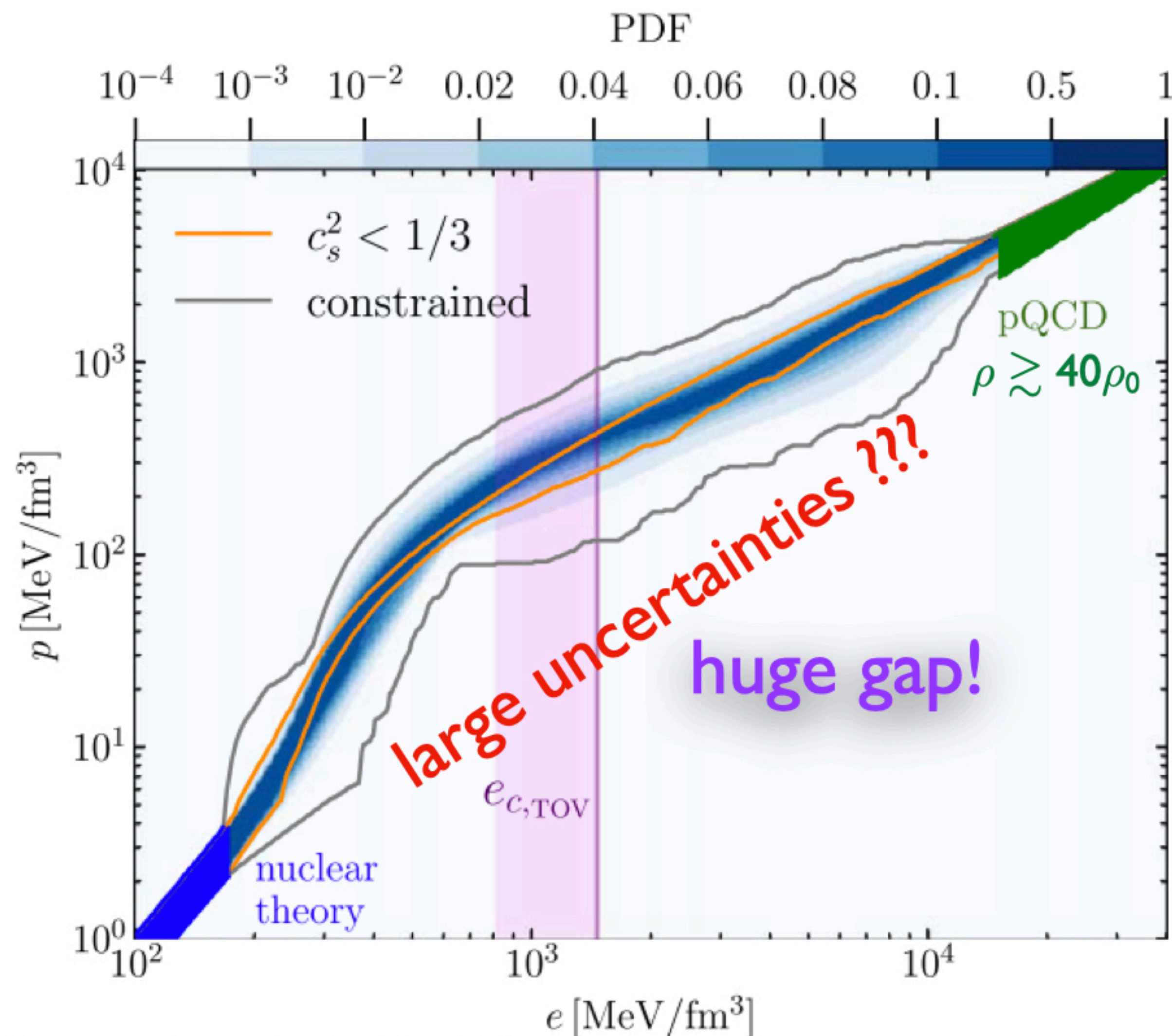
black holes : $M = 2R \rightarrow \xi_{\text{BH}} = 1/2$

white dwarfs : $\xi \sim 10^{-4}$; solar core : $\xi \sim 10^{-6}$

****core of NSs at the TOV configuration contains the densest visible matter existing in our universe**



The state-of-art constraints on cold/dense matter EOS



(1) **low-density nuclear many-body theories**

(2) **pQCD at extremely high densities**

(3) observational data (GW170817, NICER, ...)

(4) heavy-ion collision data

(5) inference algorithms (Bayesian, ML)

(6) others

$$T = 0 : \text{EOS} \rightarrow P = P(\varepsilon)$$

NS mass distribution

J1614-2230 : $1.91 \pm 0.02 M_{\odot}$



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J0348+0432 : $2.01 \pm 0.04 M_{\odot}$

Paulo Freire

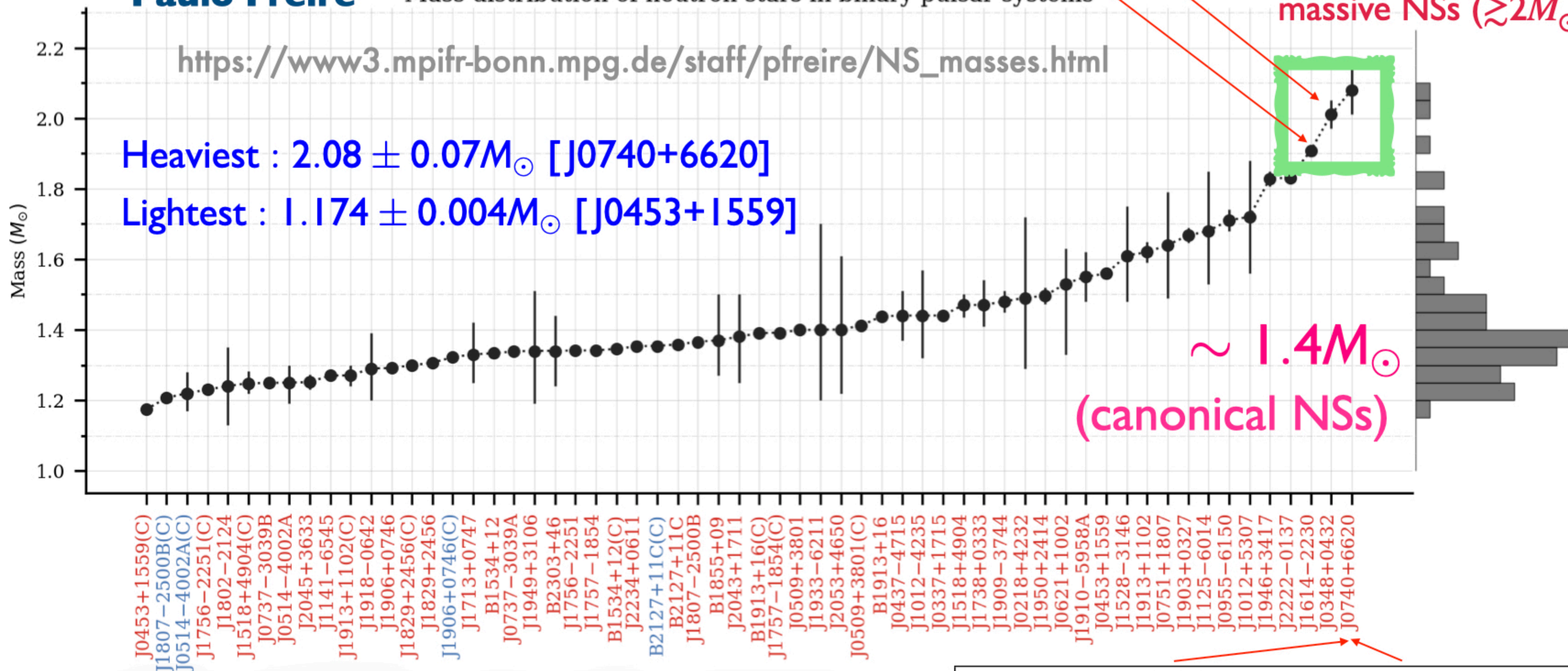
Mass distribution of neutron stars in binary pulsar systems

https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Heaviest : $2.08 \pm 0.07 M_{\odot}$ [J0740+6620]

Lightest : $1.174 \pm 0.004 M_{\odot}$ [J0453+1559]

massive NSs ($\gtrsim 2 M_{\odot}$)



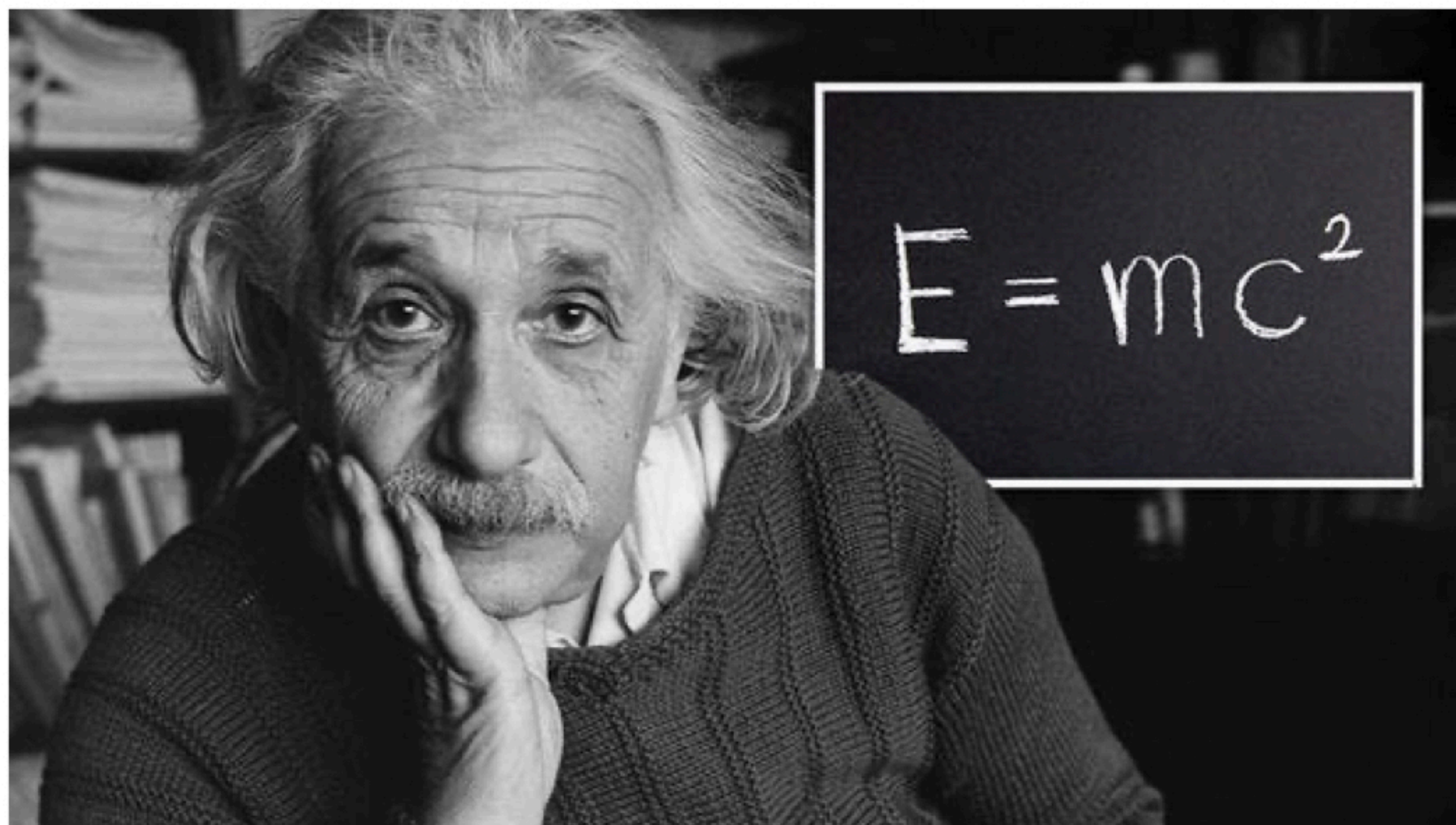
massive NSs: soft EOSs are basically excluded

$$M \approx 2.08^{+0.07}_{-0.07} M_{\odot}; R \approx 12.39^{+1.30}_{-0.98} \text{ km}$$

Is “ $\text{pressure} \leq \text{energy density}$ ” sufficient?

speed of light is the upper limit of any signal (special relativity/causality):

$$s^2 = dP/d\varepsilon \leq 1 \leftrightarrow P \leq \varepsilon$$



- (1) Can strong-field gravity in General Relativity (GR) tell us a more refined upper limit on P/ε ?
- (2) What's this upper limit?

$$\frac{d\hat{P}}{d\hat{r}} = -\frac{\hat{\epsilon}\hat{M}}{\hat{r}^2} \frac{(1 + \hat{P}/\hat{\epsilon})(1 + \hat{r}^3\hat{P}/\hat{M})}{1 - 2\hat{M}/\hat{r}}, \quad \frac{d\hat{M}}{d\hat{r}} = \hat{r}^2\hat{\epsilon},$$

Novel Scalings of Neutron Star Properties from Analyzing Dimensionless Tolman–Oppenheimer–Volkoff Equations

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Abstract

The Tolman–Oppenheimer–Volkoff (TOV) equations govern the radial evolution of pressure and energy density in static neutron stars (NSs) in hydrodynamical equilibrium. Using the reduced pressure and energy density with respect to the NS central energy density, the original TOV equations can be recast into dimensionless forms. While the traditionally used integral approach for solving the original TOV equations require an input nuclear Equation of State (EOS), the dimensionless TOV equations can be anatomized by using the reduced pressure and energy density as polynomials of the reduced radial coordinate without using any input nuclear EOS. It has been shown in several of our recent works that interesting and novel perspectives about NS core EOS can be extracted directly from NS observables by using the latter approach. Our approach is based on intrinsic and perturbative analyses of the dimensionless (IPAD) TOV equations (IPAD-TOV). In this review article, we first discuss the length and energy density scales of NSs as well as the dimensionless TOV equations for scaled variables and their perturbative solutions near NS cores. We then review several new insights into NS physics gained from solving perturbatively the scaled TOV equations. Whenever appropriate, comparisons with the traditional approach from solving the original TOV equations will be made. In particular, we first show that the nonlinearity of the TOV equations basically excludes a linear EOS for dense matter in NS cores. We then show that perturbative analyses of the scaled TOV equations enable us to reveal novel scalings of the NS mass, radius and the compactness with certain combinations of the NS central pressure and energy density. Thus, observational data on either mass, radius or compactness can be used to constrain directly the core EOS of NS matter independent of the still very uncertain nuclear EOS models. As examples, the EOS of the densest visible matter in our Universe before the most massive neutron stars collapse into black holes (BHs) as well as the central EOS of a canonical or a 2.1 solar mass NS are extracted without using any nuclear EOS model. In addition, we show that causality in NSs sets an upper bound of about 0.374 for the ratio of pressure over energy density and correspondingly a lower limit for trace anomaly in supra-dense matter. We also demonstrate that the strong-field gravity plays a fundamental role in extruding a peak in the density/radius profile of the speed of sound squared (SSS) in massive NS cores independent of the nuclear EOS. Finally, some future perspectives of NS research using the new approach reviewed here by solving perturbatively the dimensionless TOV equations are outlined.

Keywords: Equation of State, Nuclear Symmetry Energy, Neutron Star, Supra-dense Matter, Tolman–Oppenheimer–Volkoff Equations, Self-gravitating, Principle of Causality, Compactness, Stiffness, Polytropic Index, Speed of Sound, Dimensionless Trace Anomaly, Peaked Structure, pQCD Conformal Limit, Newtonian Limit, Mass-radius Relation, Causality Boundary, Strong-field Gravity, Maximum-mass Configuration, Ratio of Pressure over Energy Density, Upper/Lower Bounds

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$$H(k) \equiv \left(\frac{\bar{\epsilon}_l}{\bar{\epsilon}_l^*} \right) = \frac{3}{4} \left(1 - \frac{1}{k} \right) + \frac{\sqrt{k^2 - 2k + 9}}{4k},$$

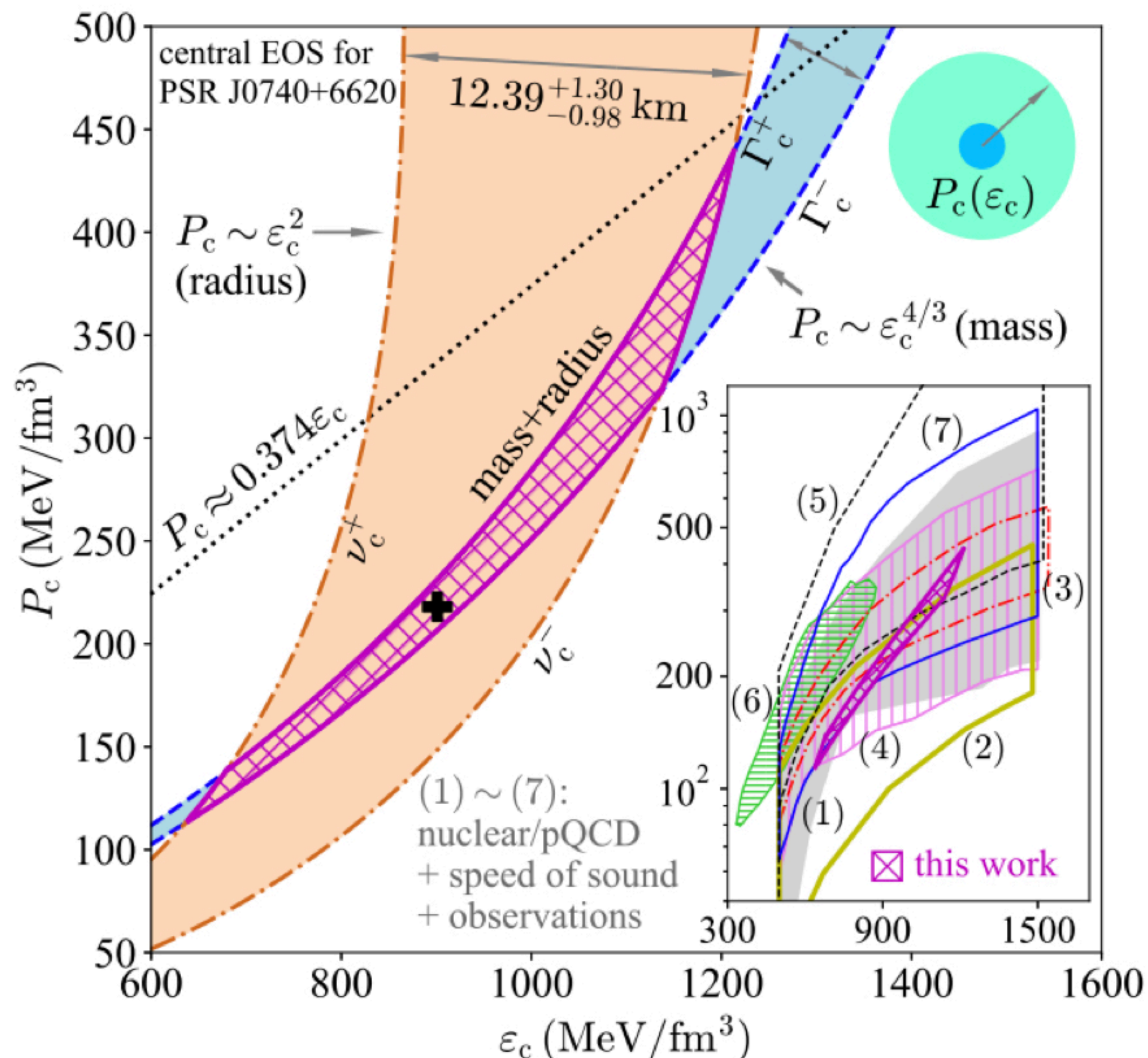
$$u_i u_j \approx 1 + \sum_{i+j \geq 1} u_i X_{ij} \mu_j^i,$$



$$s_c \sim X^{k/2} \sim (M_{\text{NS}}^{\text{max}}/R_{\text{max}})^{k/2q} \approx (M_{\text{NS}}^{\text{max}}/R_{\text{max}})^2, \quad \text{with } q \approx 1/2, k \approx 2.$$

$$P_c \sim (M_{\text{NS}}^{\text{max}})^{1+q^{-1}} R_{\text{max}}^{-3-q^{-1}} \sim M_{\text{NS}}^{\text{max},3}/R_{\text{max}}^5, \quad \text{with } q \approx 1/2.$$

Central EOS of PSR J0740+6620



$$s_c^2 = X \left(1 + \frac{1}{3} \frac{1 + 3X^2 + 4X}{1 - 3X^2} \right)$$

$$P_c \approx 218^{+93}_{-125} \text{ MeV/fm}^3; \varepsilon_c \approx 902^{+214}_{-287} \text{ MeV/fm}^3$$

$$X = P_c/\varepsilon_c \approx 0.24^{+0.05}_{-0.07} \rightarrow s_c^2 \approx 0.45^{+0.14}_{-0.18} > 1/3$$

causality requires $s_c^2 \leq 1$

$\rightarrow$ upper limit for $X = P_c/\varepsilon_c \lesssim 0.374$

(1) $s_c^2 \neq 0 \leftrightarrow$ no sharp phase transitions near center

(2) $\gamma_c = d \ln P_c / d \ln \varepsilon_c = s_c^2 / (P_c/\varepsilon_c) \geq 4/3$

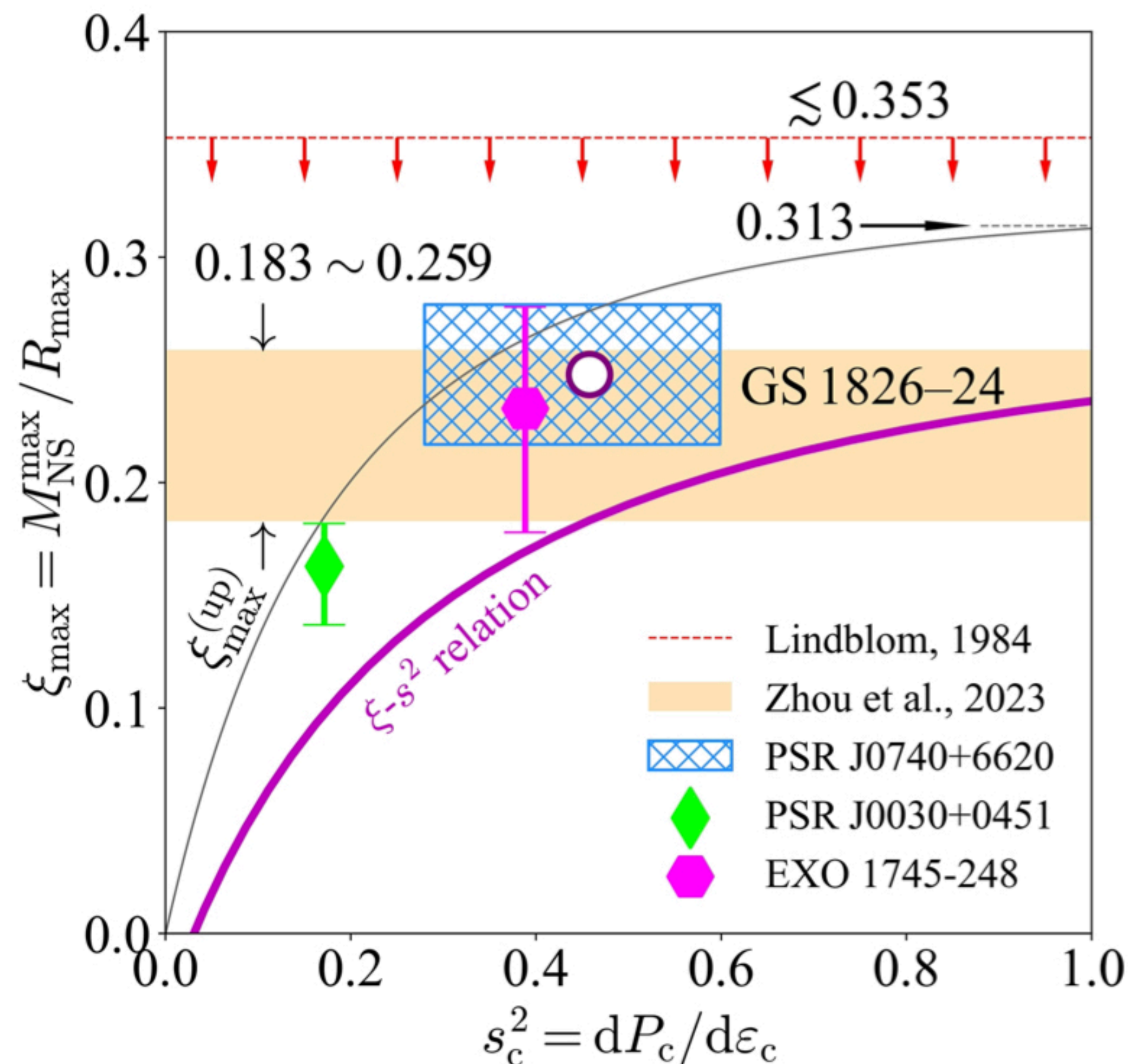
$\leftrightarrow$ central matter could not be conformal

(3) $\Delta \equiv 1/3 - P/\varepsilon \gtrsim -0.04$

IMP mass measurements: strong implications

IMP: $s_c^2 \gtrsim 0.4 \rightarrow$ exceeding the conformal bound

1/3



nature physics

$$0.183 \lesssim \xi \lesssim 0.259$$



Article

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Mass measurements show slowdown of rapid proton capture process at waiting-point nucleus ^{64}Ge

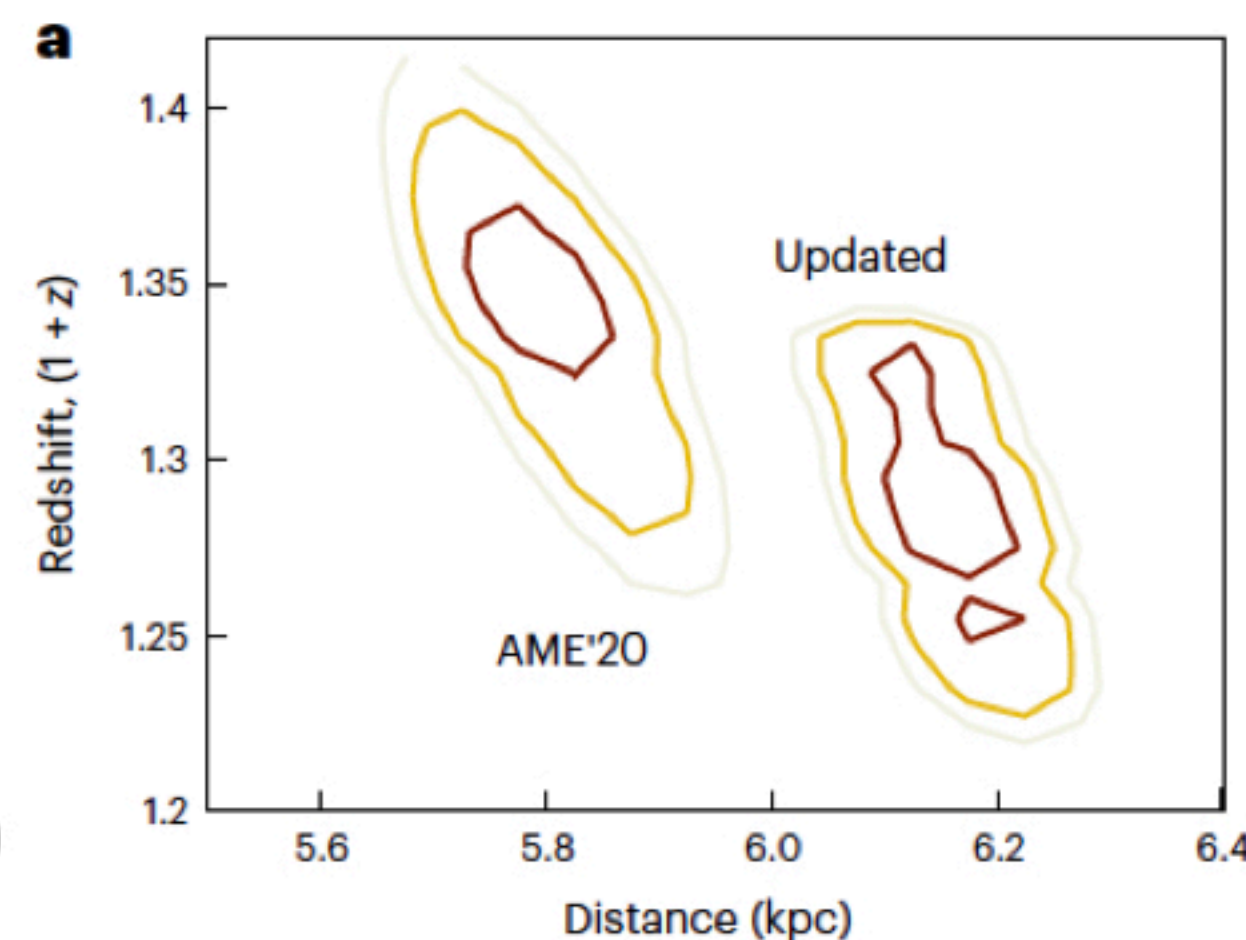
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X-ray bursts are among the brightest stellar objects frequently observed in the sky by space-based telescopes. A type-I X-ray burst is understood as a violent thermonuclear explosion on the surface of a neutron star, accreting matter from a companion star in a binary system. The bursts are powered by a nuclear reaction sequence known as the rapid proton capture process (rp process), which involves hundreds of exotic neutron-deficient nuclides. At so-called waiting-point nuclides, the process stalls until a slower β^+ decay enables a bypass. One of the handful of rp process waiting-point nuclides is ^{64}Ge , which plays a decisive role in matter flow and therefore the produced X-ray flux. Here we report precision measurements of the masses of ^{63}Ge , $^{64,65}\text{As}$ and $^{66,67}\text{Se}$ —the relevant nuclear masses around the waiting-point ^{64}Ge —and use them as inputs for X-ray burst model calculations. We obtain the X-ray burst light curve to constrain the neutron-star compactness, and suggest that the distance to the X-ray burster GS 1826-24 needs to be increased by about 6.5% to match astronomical observations. The nucleosynthesis results affect the thermal structure of accreting neutron stars, which will subsequently modify the calculations of associated observables.

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